Modeling Hourly European Electricity Spot Prices via a SARMA-GARCH Approach†

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Abstract

In the last few decades, the process of deregulation has brought changes to European electricity industries. Wholesale markets for electricity have been introduced, and the need for modeling electricity prices has become an important aspect of risk management. Consequently, decision makers require an adequate representation of uncertainty to develop hedging strategies. The aim of this work is to model hourly spot electricity prices in Austria and Spain to obtain weekly forecasts to be used for subsequent risk management as part of a stochastic programming framework. A SARMA-GARCH model is obtained for each market and presents satisfactory results in terms of adjustment and forecast performance. We propose some alternative models for comparison purposes and find that a simple SARMA model presents better forecast performance on average for the analyzed periods.

Keywords: forecasting; electricity prices; spot market; Austria; Spain

† This is a working paper produced on April 10, 2013. The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007–2013) under grant agreement no. 260041 for Collaborative Project "Energy Efficiency and Risk Management in Public Buildings" (EnRiMa). Feedback from Emilio L. Cano (Universidad Rey Juan Carlos), Michal Kaut (SINTEF), and Aron Larsson (Stockholm University) has helped to improve this work. All remaining errors are the authors' own.

1. Introduction

In the last few decades, the process of deregulation has brought sweeping changes to European electricity markets. The electricity industry used to be state regulated until the early 1990s due to its “natural monopoly” characteristics, which necessitated tight coordination between generation and transmission. However, when structural reforms started to take place worldwide motivated by the desire to improve economic efficiency, vertically integrated monopolies were partitioned into potentially competitive generation and retailing sectors with transmission and distribution functions remaining under the auspices of regulated entities. The resulting deregulated paradigm uses market prices to coordinate electricity generation and transmission in a decentralized manner and relies upon derivatives markets to facilitate risk management. Since wholesale prices are highly volatile, they require appropriate analysis along with sophisticated modeling of decisions for subsequent risk management, which was not necessary in the era of vertical integration. Wilson (2002) examines economic issues that have emerged in the context of electricity industry deregulation including the focus on the extent of reliance on markets, the detailed design of forward and spot markets, and the allocation of risks. Hyman (2010) expounds on the latter theme by positing that restructuring has effectively shifted risk onto consumers. Hence, combined with the recent impetus for transitioning to a more sustainable energy system, deregulation requires not only large power companies but also energy consumers at the building level to seek enhanced decision support in order improve energy efficiency without incurring exorbitant costs or risks.

Modeling spot prices is a key element of decision making and strategic planning. Depending on the temporal horizon, the importance of the study of electricity pricing behavior could be related to profitability and planning analysis, to derivatives contracts' pricing, or to short-term forecasts (Serati *et al.*, 2007). The need for electricity price modeling has, thus, become an important aspect of risk management in this industry for generators, retailers, and consumers as well as a source of information for government policymakers.

Motivated by this background, our work presents time-series models to analyze hourly spot electricity prices in Austria and Spain, which could be used for subsequent scenario generation as part of a stochastic-programming-based approach to decision making under uncertainty (Conejo *et al.*, 2010). Ultimately, a decision-support system (DSS) that enables building owners to make efficiency-improving retrofits to their structures and to install new energy equipment would require a means of assessing the performance of these investments. While owners of small-scale facilities are not likely to have the expertise to conduct such analyses, a developer of such a DSS would require them as inputs. Consequently, an engineering-economic analysis as in Kumbaroğlu and Madlener (2012) could be carried out using the price forecasts. Our objective here is to develop, fit, and test statistical models of electricity prices, obtain short-term forecasts for one week-ahead periods, and provide information to facilitate the development of such a DSS. We propose a combination of seasonal autoregressive moving-average (SARMA) and generalized autoregressive conditional heteroskedasticity (GARCH) models based on an entire hourly time series, using a more parsimonious model for the autoregressive part and a longer sample period than the ones presented by Contreras *et al.* (2003) and Garcia *et al.* (2005). SARMA attempts to capture the linear relationship between actual and past values in the time series, besides the seasonal pattern. Additionally, GARCH models the volatility characteristics present in financial time series. This forecasting model could be used together with an optimization tool in order to form the basis of a

DSS that small-scale consumers could use to make robust strategic or operational decisions under uncertain prices.

Unlike other commodities, electricity is distinctive because of its limited storability and transportability. These characteristics are reflected in electricity price time series, which present stylized facts that cannot be completely described by models developed for other storable commodities or financial securities. For example, Serati *et al.* (2007) and Härdle and Trück (2010) describe the observed characteristics:

1. Seasonality on an annual, weekly, and hourly basis

   Different types of seasonality can be detected in electricity prices: annual, related to the seasons during the year and to the economic and social activities during different months; weekly, related to working days and weekends; and intraday cycles, related to variations among different hours of the day.

2. Mean reversion

   Mean reversion is also observed, which may reflect long-run average costs of generation (Koopman *et al.*, 2007). Many studies have analyzed this feature in financial and commodities markets, e.g., in electricity markets (Lucia and Schwartz, 2002; Huisman *et al.*, 2007).

3. Extreme volatility

   Electricity prices exhibit high volatility in both hourly and daily data. The non-storable nature of electricity means that its price volatility is much higher than that for any other traditional commodities and that for financial markets. According to Serati *et al.* (2007) and Härdle and Trück (2010), daily price volatility can be as high as 50% compared to 3% for oil and 5% for gas (Serati *et al.*, 2007). As is typical in financial markets, volatility clusters can be observed when analyzing return time series, i.e., volatility may be high for certain periods and low for others (Tsay, 2010), thereby suggesting time-varying conditional volatility.

4. Price jumps and spikes

   In electricity prices, spikes can be caused both by supply- and demand-side shocks, e.g., generation outages and occurrence of extreme loads. Spikes can be attributed to the low level of flexibility in energy markets, determined by the non-storability of power and by the high dependence on local and temporal supply and demand conditions (Serati *et al.*, 2007). Network capacity constraints also exacerbate this effect (Cuaresma *et al.*, 2004).

There is another phenomenon specific to electricity markets: negative and zero prices. Generally, negative prices occur for only a short period and mainly at night. They can happen, for instance, because inflexible generators, e.g., nuclear power plants or district heating facilities, are too costly to shut down, thereby causing an imbalance during night hours (Sewalt and de Jong, 2003). This situation becomes a problem when modeling prices and should receive proper treatment, e.g., by either simply excluding these observations or shifting prices to zero level or working with a transformation that deals with negative prices (Schneider, 2011). The presence of renewable sources and cogeneration also reduces prices because of the effects of public support policies. This information would be important as explanatory variables to improve forecasting models (Gelabert *et al.*, 2011).

In this work, our first aim is to obtain parsimonious models for representative European electricity markets. We focus on Austria and Spain since they would allow us to examine prices in two different climate regions, i.e., Continental and Maritime, respectively. Furthermore, although the literature is more focused on multiple pools as in U.S. and Nordic Power

Exchanges, there are some papers analyzing Spanish market but not many focusing on the Austrian market. Second, the intraday seasonal pattern is modeled stochastically using SARMA for this periodicity. A SARMA model with terms of higher orders in the seasonal part is analyzed to take into account the weekly seasonality. Third, to capture the conditional volatility heteroskedasticity observed in the series, we specify a GARCH model to be combined with the SARMA. Therefore, a SARMA-GARCH methodology is used to model hourly electricity prices. The linear dependence is treated by a SARMA model using Box and Jenkins (1970), while the conditional volatility heteroskedasticity behavior is captured by a GARCH model proposed by Bollerslev (1986). The adjusted models for both countries are very similar, even though the data for each country present different characteristics of volatility, i.e., the Austrian data are more volatile than the Spanish. Due to the volatility characteristics of the time series, the SARMA-GARCH model fits the data better than the simple SARMA one. Finally, we compare SARMA and SARMA-GARCH models in terms of forecasting for both countries, but there is no gain from using the GARCH model , although the SARMA-GARCH is the best-fit model from the in-sample data. On average for the analyzed forecasts, a simple SARMA model presents slightly better performance in terms of forecast errors than SARMA-GARCH for Spain. For Austria, the difference is greater, which can be related to higher volatility in this country's electricity market compared to Spain. The rest of this paper is structured as follows. Section 2 presents a literature review of the extant time-series models used for electricity prices. Section 3 describes the methodology used in this paper. Section 4 summarizes the data, and Section 5 presents the results of the time-series analyses. Section 6 provides the conclusions, discusses the work's limitations, and provides directions for future research in this area.

2. Literature Review

The literature on electricity price modeling is very rich. Different models have been proposed based on the aforementioned observed stylized facts. According to Serati *et al.* (2007), considering the methodologies used in this field of research, three main classes can be built: autoregressive models, jump-diffusion and regime-switching models, and volatility models. The authors mention that although many papers on this subject have been published, there is not one specific model supported by empirical evidence. In fact, the suitability of models depends on the nature of the markets as well as on the scope of the underlying decision-making problem. The models differ in the data frequency used (usually daily or hourly), time-series transformation (usually logarithms of prices or log-returns), the treatment of seasonality (deterministic or stochastic), and the techniques used including continuous stochastic processes, discrete time-series models, or alternative models to handle with the aforementioned characteristics. Serati *et al.* (2007) and Higgs and Worthington (2008) present an extensive survey of the existent literature. The representative papers mentioned in this work can be summarized as follows in Table 1.

Table 1 – Papers and main results

Paper	*Market*	*Frequency*	*Model*	*Seasonality treatment*	*Main results*
Lucia and Schwartz (2002)	Nordic electricity market	Daily	One-factor mean-reverting stochastic process or two-factor stochastic process (mean-reverting process and GBM)	Deterministic function	The modeling is done for the purposes of pricing derivatives. Seasonal patterns are crucial in explaining the term structure of futures prices at the Nord Pool. Accurate estimation of seasonal pattern as the annual frequency requires large time series. The volatility is consistently different between cold and warm seasons.

Contreras *et al.* (2003)	Spanish and Californian electricity markets	Hourly	ARIMA models	Seasonal patterns considered in high order ARMA terms	The paper uses ARIMA models. Spanish market shows more volatility in general and needs data from the previous five hours. In California market, the model needs data from the previous two hours.
Garcia *et al.* (2005)	Spanish and Californian electricity markets	Hourly	AR models of high orders + GARCH	Seasonal patterns considered in high-order AR terms	The authors propose an AR-GARCH model to forecast using AR terms of very high orders (more than lag 500) and GARCH (1,3). The results are better than using an ARIMA model. Adding demand as an explanatory variable improves the forecast.
Higgs and Worthington (2005)	Australian electricity market	Half-hour data	AR(1) + GARCH model and variations	Seasonal treatment in the volatility process	The paper investigates intraday price volatility process. The asymmetric skewed Student APARCH specification is the one that produces the best results in most cases analyzed. Significant asymmetric responses are also observed.
Thomas and Mitchell (2005)	Australian electricity market	Half-hour data	ARMA-GARCH model and variations (TARCH, EGARCH and PARCH)	Deterministic function	The authors compare the efficiency of four different GARCH model specifications to describe volatility processes by incorporating seasonal effects and spikes in the conditional mean. Significant ARCH effects are observed, and asymmetric volatility response is detected. The estimated GED parameter confirms fat-tailed properties.
Weron and Misiorek (2005)	Californian power market	Hourly	ARMA and ARMAX (including system loads and plant data as exogenous variable)	Dummies to account for weekly seasonality. Seasonal patterns also considered in high order AR terms	The authors aim to study simple time-series models and assess their forecast performance. The best results are obtained for pure ARX models, with lagged terms in 24, 48, and 168. Dummies are used for Mondays, Saturdays, and Sundays
Huisman *et al.* (2007)	Dutch, German, and French wholesale power markets	Hourly	Stochastic component modeled as mean-reverting process	Deterministic function	The authors argue that dynamics of hourly prices do not behave as a time-series process and propose a panel model for hourly electricity prices. Each hourly series

					exhibits hourly specific mean-reversion parameters. Prices in peak-hours are highly correlated among each other (the same is valid for off-peak hours)
Karakatsani and Bunn (2008)	British energy market	Half-hour data	Time-varying coefficients regression model	Trigonometric function with time-varying coefficients	The authors propose a regression model to allow for a continuously adaptive price structure. The drivers are demand level, slope, curvature and volatility, margin, scarcity, learning, spread, seasonality, and trend. Each half-hour price time series is modeled separately. They also propose a regime-switching approach as an extension and compare some alternative models with a simple AR model. Price models representing market fundamentals and time-varying effects exhibit better forecasting performance.
Ming *et al.* (2008)	American PJM market	Hourly	SARIMA model +GARCH	Seasonal patterns considered in high-order AR terms	The authors propose to divide the constant day series into working-day series and holiday series. SARIMA+GARCH models are established separately. This approach increases the forecasting precision in the presented example.
Bisaglia *et al.* (2010)	French, Austrian, and Spanish electricity markets	Hourly	Autoregressive-GARCH and Markov-switching models	Deterministic function	The authors extend a model under panel framework specifying a GARCH structure, introducing cross-lagged correlation through VAR-type specification and spikes through Markov regime-switching models. They obtain improvements from GARCH in the day-ahead forecast and from VAR and MS models in longer horizons.
Heydari and Siddiqui (2010)	British energy markets	Daily	Different linear stochastic processes (including mean-reverting process, ABM and GBM) and non-linear stochastic models to account for the spikes	Deterministic function	The objective is valuation of a gas-fired power plant. Among linear stochastic models, the mean-reverting one is the best-fit and presents the best out-of-sample forecasting performance. Taking into account the existence of spikes, the non-linear models

					provide more accurate long-term decision-making, even though leading to large RMSEs when compared to historical data.
Montero *et al.* (2011)	Spanish electricity market	Daily	T-ARSV and GARCH variations	Deterministic function	The authors focus on modeling asymmetric patterns of the volatility of electricity spot prices. They compare a T-ARSV (threshold autoregressive stochastic volatility model) with four GARCH-type specifications. Asymmetric responses have been detected as a traditional leverage effect.
Peña (2012)	German, French, and Spanish markets	Hourly	Periodic autoregressive models	Dummies for weekdays and stochastic seasonality treated in periodic autoregressive elements	The author proposes an autoregressive periodic panel model, using an individual model for each hour of the day. For individual prices, the autoregressive periodic model present better results than standard auto-regressive mean-reverting processes. Moreover, modeling all hourly prices jointly as a panel, periodic components models fit data better than non-periodic models.

Continuous stochastic processes have also been used to model the main characteristics of electricity prices such as mean reversion and spikes. Lucia and Schwartz (2002) express daily spot prices and logarithms of spot prices as a sum of two components: a deterministic part to model the seasonality and a stochastic part, for which they propose either a one-factor mean-reverting stochastic process or a two-factor stochastic process combining a mean-reverting process and geometric Brownian motion (GBM) to model the correlation between spot and futures prices. Heydari and Siddiqui (2010) also propose the decomposition of logarithms of prices by modeling the stochastic part as different linear stochastic processes and using non-linear stochastic models to account for the spikes. Huisman *et al.* (2007) work with separate hourly prices series, one for each hour of the day, and propose the same decomposition for all hours (into a deterministic seasonal function and stochastic part, which is modeled as a mean-reverting process). They identify mean-reversion parameters for each hour of the day.

Traditional time-series models are also frequent in the literature, based on autoregressive moving-average (ARMA) Box-Jenkins models. Moreover, since time-varying conditional volatility is shown to be a stylized fact in electricity price time-series, many authors propose the application of GARCH models as shown in Table 1. Bisaglia *et al.* (2010) also decompose hourly prices into a deterministic part, to model the seasonality, and a stochastic part, described by autoregressive-GARCH and Markov-switching models. Weron and Misiorek (2005) study simple ARMA and ARMAX (including system loads as exogenous variables) models using logarithmic transformation for hourly prices. The dependence in the AR part is related to observations in lags 24, 48, and 168. They also use weekly seasonal dummies to account for seasonality as a deterministic function. Contreras *et al.* (2003) use ARIMA models, and Garcia *et al.* (2005) find that an AR-GARCH model forecasts hourly electricity prices better than an ARMA one. In both cases, the AR part is modeled with very high orders, i.e., considering lags greater than 500, in order to capture seasonal patterns. On the other hand, Ming *et al.* (2008) propose a SARIMA-GARCH for logarithms of prices with lower orders, i.e., the autoregressive part includes lags until 27. Peña (2012) alternatively proposes a periodic autoregressive model for each hourly series, i.e., one for each hour of the day, as well as modeling hourly prices jointly as a panel. His results suggest that autoregressive models with periodic components better describe hourly series than standard autoregressive models. Moreover, he recommends using the panel with all twenty-four hourly prices as the underlying process instead of the prices at a specific hour. Thomas and Mitchell (2005) analyze the five regions of Australia's National Electricity Market using a six-year sample of high-frequency half-hourly time series, which is broader than previous work for Australia. They argue that a larger database is necessary to understand better the volatility process. They use dummy variables to treat seasonality, spikes, and negative prices. Afterwards, basic GARCH-type models are tested (GARCH, exponential GARCH, threshold GARCH, and power ARCH) to consider the evidence of asymmetric effects in the volatility process as also observed by Higgs and Worthington (2005) and Montero *et al.* (2011). Thomas and Mitchell (2005) mention the difficulty of attaining the wide-sense stationarity condition of the GARCH model given by a constraint in the coefficients' values of the model. Another feature is that a non-Gaussian distribution for conditional error terms is used. Other alternative models to explain and forecast electricity prices are also present in the literature, such as the time-varying coefficients regression model presented by Karakatsani and Bunn (2008), where the price depends on fundamental factors or drivers.

Our interest is in handling high-frequency data, i.e., hourly electricity prices. In the European Energy Exchange (EEX), for example, the spot price is an hourly contract with physical delivery, and each day is divided into twenty-four hourly contracts (Härdle and Trück, 2010). In the day-ahead markets, prices for all hours of the next day are determined at the same time. Some authors model each hour time-series separately, while others treat as an entire time series in sequence. There are some arguments that hourly prices cannot be treated as a pure time-series process because of the specific structure of day-ahead markets (Härdle and Trück, 2004; Huisman *et al.*, 2007). Huisman *et al.* (2007) and Peña (2012) propose a panel model, thereby resulting in one model for each hour of the day and a cross-sectional correlation matrix. Karakatsani and Bunn (2008) also propose modeling each intraday trading period separately due to distinct price profiles. On the other hand, many authors present models using an entire twenty-four-hour series, where observations are taken in sequence (Contreras *et al.*, 2003; Weron and Misiorek, 2005; Thomas and Mitchell, 2005). An advantage in this case is that it is possible to work with only one model for every hour in a day, and the correlation between the hours can be treated within the same model. Cuaresma *et al.* (2004) present models based on both approaches. Using data from the Leipzig Power Exchange, they obtain better forecasting properties when modeling each hour separately. Sewalt and de Jong (2003) mention that when modeling hourly prices, it is important to capture the interdependencies in different hours during the same day and between equivalent hours on different days.

Summarizing the applications presented in several papers, which analyze markets in the US and Europe, it can be said that the time span used for both in-sample (to estimate unknown parameters) and out-of-sample (to assess forecasting performance) periods varies a lot. As for the forecast period, up to one week forecasts are normally chosen. Given this background, the contribution of this paper is twofold. First, we propose a parsimonious model, which is easy to implement, in order to obtain hourly prices forecasts for a week, using high-frequency data of five years in Spanish and Austrian markets, based on hourly time series. Second, we compare some models to analyze the benefits in terms of adjustment and forecasting of combining a GARCH model with a simple SARMA one and of using high-order terms in the seasonal part to account for intraday and weekly seasonality. The resulting forecasting model would be appropriate to use as an input to an operational or strategic DSS.

3. Methodology: SARMA-GARCH Model Approach

As is customary in electricity price modeling, in order to obtain a more stable variance, we propose working with the logarithmic transformation, i.e.:

$$p_t = log\, P_t \tag{1}$$

where P_t is the spot electricity price for hour *t*. A traditional time-series analysis begins by fitting an ARMA*(p,q)* model and examining the behavior of the error term. A general model is given by:

$$\Phi_p(L)y_t = \Theta_q(L)\epsilon_t \quad (2)$$

where L is the lag operator, $\Phi_p(L)$ and $\Theta_q(L)$ are polynomials of degrees p and q, respectively, ϵ_t is a N $(0,\sigma^2)$ disturbance term, and y_t is a stationary time series. More specifically, we have the following polynomials:

$$\Phi_p(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p \quad (3)$$

and

$$\Theta_q(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q \quad (4)$$

Since it is desirable for y_t to be a stationary series, we normally work with log-return series, which is equivalent to obtaining the first difference of the logarithmic series. It is common to assume that returns are weakly stationary, and this characteristic can be checked through statistical tests, e.g., unit root tests (Tsay, 2010). When working with electricity prices, it may be necessary to consider seasonal stationarity. In this case, we need to take differences in seasonal periods as well.

A linear time-series model can be characterized by its autocorrelation function (ACF), i.e., the cross-correlation of a series. Modeling it makes use of the sample ACF to capture the linear dynamics of the data. The sample ACF calculates the autocorrelation for different lags providing information about linear dependence and for model identification. For financial time series, in general, an AR*(p)* term is enough to capture the linear dependence. The order *(p,q)* of an ARMA model in financial applications depends on the frequency of the series and is defined from ACF and partial ACF (PACF) analyses. For electricity prices, it is necessary to use a seasonal ARMA model to capture the seasonality.

Also, it is well known that although volatility is not directly observable, there are some specific characteristics observed in financial returns time series, such as the volatility-clustering effect (Tsay, 2010). After fitting an ARMA model, the presence of volatility clusters suggests that, besides modeling the linear dependence, it is necessary to study the volatility, for which a GARCH model is proposed in conjunction with a SARMA one. The conditional variance is given by:

$$\mathrm{E}[{\epsilon_t}^2 | I_{t-1}] = \sigma_t^2 \tag{5}$$

where I_{t-1} is the information available until time t-1. The residuals for Eq. (2) and the variance equation can be written as:

$$\epsilon_t = \sigma_t \eta_t \tag{6}$$

$$\sigma_t^2 = \alpha_o + \sum_{i=1}^{m} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{7}$$

where $\eta_t \sim \mathrm{N}(0,1)\ i.i.d.$ and $\sum_{i=1}^{m} \alpha_i + \sum_{j=1}^{s} \beta_j < 1$ to guarantee that unconditional variance is finite.

In order to obtain the parameters of the models and to analyze the forecast performance, the dataset is split into two periods. Using an in-sample period, the unknown parameters for the models are estimated. The forecast is obtained and analyzed for the out-of-sample period. Under this framework, SARMA and SARMA-GARCH models are proposed based on the data series' characteristics. The best model is chosen considering the AIC (Akaike information criterion) and BIC (Bayesian information criterion, also known as the Schwarz criterion), which are measures of the relative goodness-of-fit of statistical models and residual diagnostics.

From the fitted models, we forecast prices in the out-of-sample period. To verify forecast performance, aggregate error measures such as the mean absolute error (MAE) and the root mean squared error (RMSE) are used for 168 steps-ahead, i.e., one week ahead. The MAE and RMSE are defined by:

$$MAE = \frac{\sum_{t=N+1}^{N+168} |e_t|}{168} \tag{8}$$

$$RMSE = \sqrt{\frac{\sum_{t=N+1}^{N+168} e_t^2}{168}} \tag{9}$$

where $e_t = P_t - \hat{P}_t$ is the forecast error, i.e., the difference between the actual price P_t and the forecasted price $\hat{P}_t$ at time *t*. The measures can be calculated for different origins of time, where *N* is the length of the observed data period and *N+1* is the first observation in the chosen week to be forecasted. We use the two aforementioned methodologies to analyze the forecast performance.

Two types of prediction methods may be used: dynamic and static using the Eviews software (Brooks, 2008). The dynamic method calculates multi-step forecasts starting from the first period in the forecast sample. Therefore, the dynamic prediction uses forecasted prices for each hour given the last observation at time *t* before the 168-hour period to be analyzed. This measure is used to analyze the forecast performance for a week-ahead period. The forecasted price at time $t+s$ is given by:

$$\hat{P}_{t+s} = \mathrm{E}[P_{t+s} \,|P_t\,] \qquad s = 1\ to\ 168 \tag{10}$$

By contrast, the static method calculates a sequence of one-step-ahead forecasts, rolling the sample forecast one after each forecast (Brooks, 2008). In this case, static prediction calculates the forecasted price given the last most updated observation, i.e., each forecast within the period of 168 hours is given one step-ahead. Since the actual values are used in each forecast, this measure can be used to verify the model adjustment. The forecasted price at time $t+s$ is given by:

$$\hat{P}_{t+s} = \mathrm{E}[P_{t+s} \,|P_{t+s-1}\,] \qquad s = 1\ to\ 168 \tag{11}$$

The present model can be applied to complete hourly time series, thereby providing one model to all hours of the day, or to twenty-four separate time series, one for each hour of the day, which will result in twenty-four different models. In this paper, we propose modeling one entire time series. Using this approach, it is possible to work with only one model for every hour in a day.

4. Electricity Hourly Prices Data

4.1. Spanish Market Data

For the Spanish market, a total of 42,720 hourly observations over five years of electricity spot prices in €/MWh are available from www.omel.es. The sample period begins on January 1, 2007 and ends on November 15, 2011. There are five missing values and 379 zeros. Besides the 384 missing/zero values, there are also very low prices in the sample. For example, 84 values that are equal or less than €1/MWh are detected. The dataset is split into two periods: in-sample (January 1, 2007 to December 31, 2010), which has 35,064 hourly observations, and out-of-sample (January 1, 2011 to November 15, 2011), which has 7,656 hourly observations. Thus, there is: (i) one entire series with 42,720 hourly observations and descriptive statistics presented in Table 2, and (ii) twenty-four separate series for each hour of the day with 1,780 daily observations. Descriptive statistics by hour are presented in Table 3.

Table 2 – Summary of descriptive statistics for Spanish electricity prices (entire series)

Statistic	*Value*
Mean (€/MWh)	45.38
Standard Deviation (€/MWh)	16.37
Variance	268.1
Skewness	0.24
Kurtosis	3.55
Number of observations	42720

Table 3 – Summary of descriptive statistics for Spanish electricity prices (twenty-four hours)

Statistic	*Value*											
	hr 1	***hr 2***	***hr 3***	***hr 4***	***hr 5***	***hr 6***	***hr 7***	***hr 8***	***hr 9***	***hr 10***	***hr 11***	***hr 12***
Mean (€/MWh)	44.79	40.25	35.88	34.03	32.59	33.69	37.80	42.70	44.76	47.22	49.98	50.18
Std. Dev. (€/MWh)	13.51	13.63	14.05	14.18	14.08	13.81	13.98	14.69	15.91	15.80	15.78	15.59
Variance	182.4	185.8	197.3	201.0	198.2	190.6	195.5	215.8	253.1	249.7	249.1	243.2
Skewness	0.14	-0.04	-0.19	-0.20	-0.18	-0.23	-0.25	0.01	0.04	0.19	0.31	0.33
Kurtosis	3.43	3.54	3.26	3.03	2.89	3.00	3.31	3.40	3.39	3.38	3.42	3.41
# of obs.	1780	1780	1780	1780	1780	1780	1780	1780	1780	1780	1780	1780
Statistic	*Value*											
	hr 13	***hr 14***	***hr 15***	***hr 16***	***hr 17***	***hr 18***	***hr 19***	***hr 20***	***hr 21***	***hr 22***	***hr 23***	***hr 24***
Mean (€/MWh)	50.90	49.76	46.74	45.61	45.37	46.85	49.66	52.75	54.16	55.70	51.11	46.71
Std. Dev. (€/MWh)	15.38	15.37	14.49	14.58	15.00	15.27	16.50	17.53	16.77	15.65	14.34	14.42
Variance	236.5	236.2	209.9	212.7	225.1	233.3	272.3	307.1	281.2	244.8	205.7	208.0
Skewness	0.33	0.29	0.24	0.10	0.10	0.20	0.56	0.57	0.58	0.68	0.52	0.27
Kurtosis	3.45	3.43	3.45	3.45	3.42	3.31	3.41	3.01	2.78	3.32	3.03	3.27
# of obs.	1780	1780	1780	1780	1780	1780	1780	1780	1780	1780	1780	1780

4.2. Austrian Market Data

For Austria, a total of 43,080 hourly observations over five years of electricity spot prices in €/MWh are available from www.exaa.at. The sample period begins on January 1, 2007 and ends on November 30, 2011. There are five missing values in addition to very low prices in the sample. For example, 184 values that are equal or less than €1/MWh were detected. The dataset is again split into two periods: in-sample (January 1, 2007 to December 31, 2010), which has 35,064 hourly observations, and out-of-sample (January 1, 2011 to November 30, 2011), which has 8,016 hourly observations. Considering the dataset provided by EXAA, there is again: (i) one entire series with 43,080 hourly observations, for which the descriptive statistics are presented in Table 4, and (ii) twenty-four separate series for each hour of the day with 1,795 daily observations, for which the descriptive statistics are presented in Table 5.

Table 4 – Summary of descriptive statistics for Austrian electricity prices (entire series)

Statistic	*Value*
Mean (€/MWh)	48.20
Standard Deviation (€/MWh)	23.92
Variance	572.32
Skewness	2.40
Kurtosis	25.67
Number of observations	43080

Table 5 – Summary of descriptive statistics for Austrian electricity prices (twenty-four series)

Statistic	*Electricity price*											
	hr 1	***hr 2***	***hr 3***	***hr 4***	***hr 5***	***hr 6***	***hr 7***	***hr 8***	***hr 9***	***hr 10***	***hr 11***	***hr 12***
Mean (€/MWh)	37.23	32.72	29.32	26.90	27.24	31.68	38.51	50.55	55.44	58.81	61.17	64.79
Std. Dev. (€/MWh)	12.37	12.34	12.41	12.31	12.47	13.40	17.50	24.16	23.75	23.74	24.29	27.30
Variance	153.0	152.3	154.1	151.6	155.6	179.6	306.4	583.8	564.2	563.5	590.2	745.1
Skewness	0.17	0.02	-0.03	0.08	0.07	-0.08	-0.01	0.57	0.88	1.31	1.60	1.97
Kurtosis	2.86	2.67	2.56	2.43	2.48	2.72	2.78	3.71	4.74	5.95	6.80	8.76
# of obs.	1795	1795	1795	1795	1795	1795	1795	1795	1795	1795	1795	1795
Statistic	*Electricity price*											

	hr 13	hr 14	hr 15	hr 16	hr 17	hr 18	hr 19	hr 20	hr 21	hr 22	hr 23	hr 24
Mean (€/MWh)	60.41	56.93	53.72	51.17	51.39	57.86	61.72	59.02	54.09	48.56	47.18	40.43
Std. Dev. (€/MWh)	22.45	21.86	21.38	20.47	21.63	34.35	33.82	23.64	18.25	14.72	13.52	12.10
Variance	503.9	478.0	456.9	418.9	468.0	1179.7	1143.7	558.9	333.1	216.5	182.9	146.4
Skewness	1.55	1.39	1.32	1.24	1.54	5.51	5.03	2.05	1.02	0.75	0.45	0.29
Kurtosis	6.60	5.92	5.66	5.32	7.18	55.83	49.25	13.57	4.75	3.87	3.20	2.99
# of obs.	1795	1795	1795	1795	1795	1795	1795	1795	1795	1795	1795	1795

Even though the Austrian data include fewer zeros and fewer very low values than the Spanish ones, the Austrian market is more volatile. In particular, its standard deviation is higher: 23.92 as opposed to 16.37 for Spanish data. When comparing the twenty-four separate hourly series in terms of volatility, the behavior of these hourly series for the Spanish market is very similar. Volatility varies from 13.51 (hour 1) to 17.53 (hour 20). On the other hand, there is considerable difference among the volatilities of the twenty-four series in Austria market, varying between 12.31 (hour 4) to 34.35 (hour 18). This information could be taken into account when modeling twenty-four hours separately.

5. SARMA-GARCH Model Results

5.1. Spain

Because of the outliers, a database treatment is performed by detecting data with extreme low outliers, e.g., points where price equals to 0, in order to obtain reliable forecasts. Considering the log-return series for each hour of the day, points more than five standard deviations away from the mean are filtered. The data related to the day of this observation are then replaced by the corresponding data of the day before. This replacement involves less than 5% of the data. Figures 1(a) and 1(b) present the original price and log return time series, respectively. Figures 2(a) and 2(b) present the treated price and log return time series for the treated data, respectively.

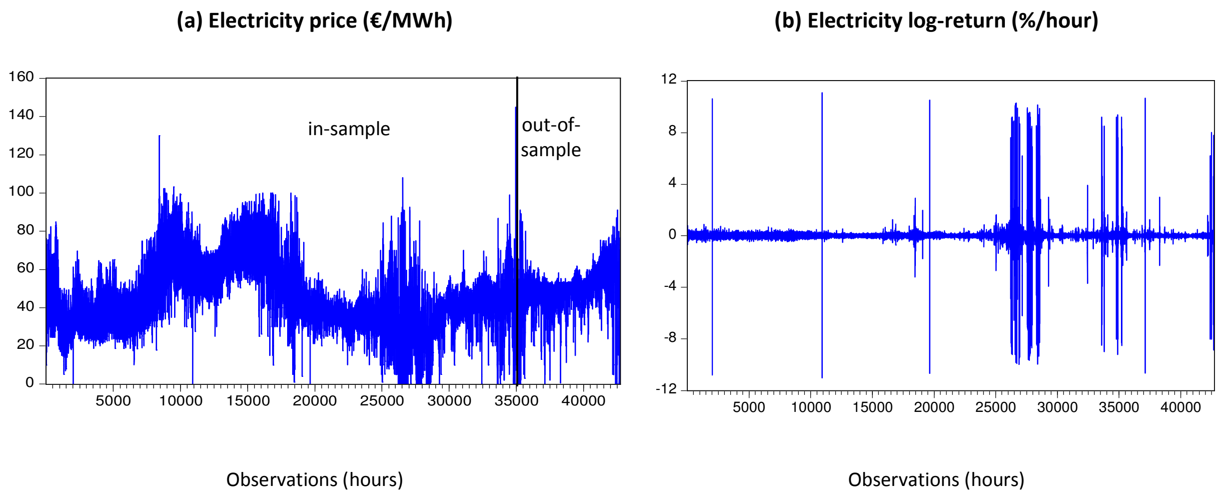


Fig. 1 - Spanish prices and log-return series

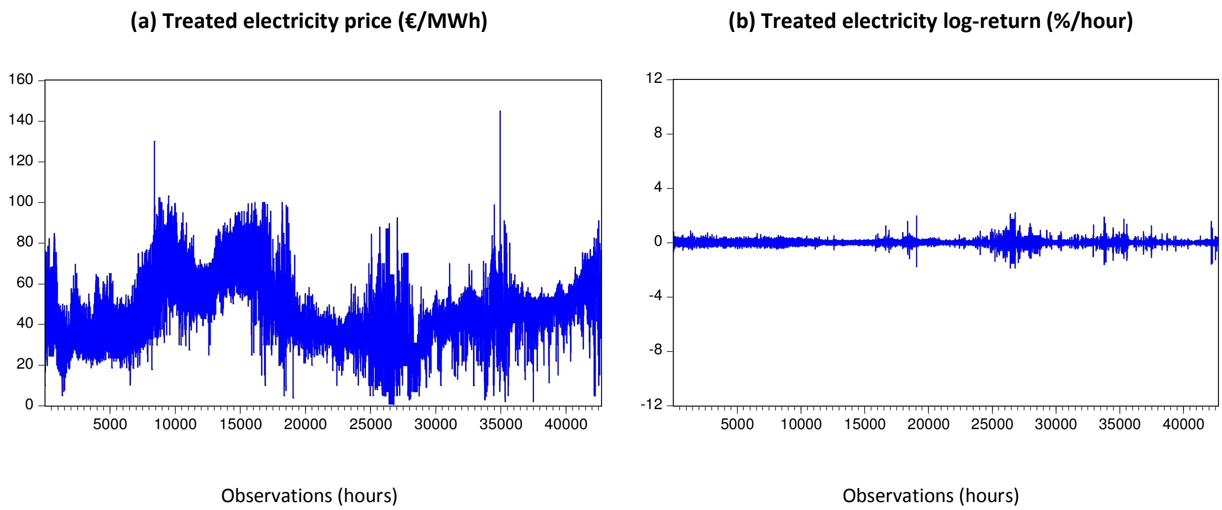


Fig. 2 - Spanish treated prices and log return series

The sample ACF for the electricity prices in Figure 3(a) illustrates the existence of spikes at lags equal to 24 and multiples, thereby revealing intraday seasonality. In order to obtain both stationarity and seasonal stationarity, a first differentiation and a periodic differentiation of order 24 for log prices are taken. Figure 3(b) presents the corresponding autocorrelation function after differentiations. Thus, the series to be modeled is given by:

$$y_t = (1 - L)(1 - L^{24}) log P_t \tag{12}$$

(a) ACF for prices **(b) ACF for log prices after differentiations**

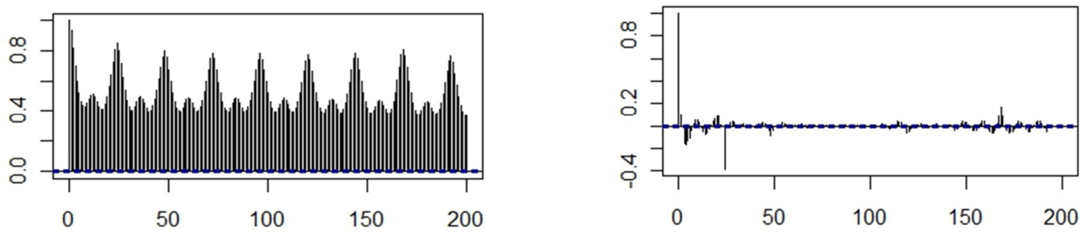


Fig. 3 – ACF for Spanish treated prices and differentiated prices

Following the methodology of modeling the linear dependence by a seasonal ARMA and the conditional variance by a GARCH model, the parameters of the models are estimated using in-sample period data, from January 1, 2007, hour 1, to December 31, 2010, hour 24. It is worth mentioning that GARCH coefficients are estimated subject to coefficient constraints in order to guarantee stationarity, as given by Equation (7).

We first analyze simple SARMA models with no conditional volatility treatment. The results are presented in Table 10 in the Appendix. Among the first models tested, the SARMA(2,2)x(1,1)$_{24}$ one is a good choice according to AIC and BIC criteria besides the log-likelihood function value. However, to treat the autocorrelation in lag 168, which is related to weekly seasonality, we test some models adding higher-order terms for the seasonal part. The best-fit model is a SARMA(2,2)x(7,7)$_{24}$. We decide to keep it parsimonious and use just the terms related to first and seventh orders for autoregressive and moving average components in the seasonal part.

When adding a GARCH model to treat the conditional volatility, the best-fit model for the series is a SARMA(2,2)x(7,7)$_{24}$+GARCH(2,2) according to AIC and BIC criteria. The estimation results are presented in Table 6 including the coefficients' values and statistical significance information. Additional information for other models tested is presented in Tables 11 and 12 in the Appendix.

Table 6 – Estimated parameters for SARMA(2,2)x(7,7)$_{24}$-GARCH(2,2) in entire hourly series

Variable	*Coefficient*	*Std. Error*	*z-Statistic*	*Prob.*
AR(1)	1.189895	0.004756	250.1797	0.0000
AR(2)	-0.330872	0.004793	-69.02608	0.0000
SAR(24)	0.183809	0.003622	50.74288	0.0000
SAR(168)	0.257128	0.003210	80.11340	0.0000
MA(1)	-1.223857	0.000218	-5607.858	0.0000
MA(2)	0.225591	0.000240	938.1322	0.0000
SMA(24)	-0.863156	0.002085	-414.0352	0.0000
SMA(168)	-0.039954	0.001910	-20.91510	0.0000
C	8.78E-06	--	--	--
ARCH(1)	0.305713	0.002182	140.0918	0.0000
ARCH(2)	-0.293537	0.002265	-129.6199	0.0000
GARCH(1)	1.426945	0.000640	2230.291	0.0000
GARCH(2)	-0.440086	0.000825	-533.1623	0.0000

The equations for the SARMA-GARCH model are:

$$(1-\varphi_1 L-\varphi_2 L^2)(1-\Phi_{24}L^{24}-\Phi_{168}L^{168})y_t = (1+\theta_1 L+\theta_2 L^2)(1+\Theta_{24}L^{24}+\Theta_{168}L^{168})\epsilon_t \quad (13)$$

$$\sigma_t^2=\alpha_0+\alpha_1\epsilon_{t-1}^2+\alpha_2\epsilon_{t-2}^2+\beta_1\sigma_{t-1}^2+\beta_2\sigma_{t-2}^2 \quad (14)$$

where

$\varphi_1 = 1.189895$ $\theta_1 = -1.223857$

$\varphi_2 = -0.330872$ $\theta_2 = 0.225591$

$\Phi_{24} = 0.183809$ $\Theta_{24} = -0.863156$

$\Phi_{168} = 0.257128$ $\Theta_{168} = -0.039954$

$\alpha_0 = 8.78\text{E} - 06$

$\alpha_1 = 0.305713$ $\beta_1 = 1.426945$

$\alpha_2 = -0.293537$ $\beta_2 = -0.440086$

Analyzing the residuals, they are not normal due to high kurtosis. The histogram for the residuals is presented in Figure 13 in the Appendix. The presence of outliers is the most important cause of this effect. Even modeling the errors as fat-tailed distributions in the GARCH model, such as Student's *t*-distribution and Generalized Error Distribution (GED), is not enough to obtain normally distributed errors. The residuals show that the seasonality is well captured by the SARMA model. As mentioned, we use seasonal AR and MA terms of order 7 in the seasonal part to take into account the weekly seasonality. We decide to keep a parsimonious model by using seasonal terms AR and MA for periods 24 and 168. As analyzed before, the corresponding SARMA $(2,2)\text{x}(7,7)_{24}$ model turns to be a better choice, according to AIC and BIC, compared to the SARMA$(2,2)\text{x}(1,1)_{24}$ one, and the same is observed when including the GARCH(2,2) model. An alternative to treating different cycles would be to include dummies to capture seasonality other than the one modeled by SARMA or to work with a double SARMA model (Mohamed *et al.*,,2011)., which extends the polynomial functions of the ARMA models to include multiple seasonal periods. Contreras *et al.* (2003) also present this possibility in a general formulation.

After having obtained the fitted model, we analyze the model adjustment and the forecast performance for the twenty-four hours of each day in the next week. Table 7 presents the MAE and RMSE measures for one week in each month of the out-of-sample year of 2011, 168 steps-ahead. The weeks for forecasting results are chosen considering periods in which there are not treated data during the week for prediction.[1] Table 6 also presents the standard deviation for each week varying from 3.95 to 11.94.

Table 7 – Forecast performance for dynamic and static forecast

Month	*Week*	*Std. deviation (€/MWh)*	*Static 168 steps-ahead forecast (€/MWh)*		*Dynamic 168 steps-ahead forecast (€/MWh)*	
			RMSE	**MAE**	**RMSE**	**MAE**
January	17/01/2011 - 23/01/2011	11.94	5.46	3.80	9.39	7.60
February	01/02/2011 - 07/02/2011	5.76	1.93	1.45	3.68	2.72
March	20/03/2011 - 26/03/2011	7.06	2.20	1.63	3.82	3.14
April	24/04/2011 - 30/04/2011	7.04	2.71	1.87	6.21	4.57
May	25/05/2011 - 31/05/2011	4.32	1.32	0.97	3.39	2.37
June	24/06/2011 - 30/06/2011	7.69	2.74	1.60	6.23	3.45
July	25/07/2011 - 31/07/2011	5.50	1.94	1.33	3.78	3.07
August	25/08/2011 - 31/08/2011	3.95	1.48	1.14	2.55	2.16
September	24/09/2011 - 30/09/2011	7.64	2.09	1.53	4.74	3.92
October	17/10/2011 - 23/10/2011	11.06	2.63	2.04	8.46	6.10

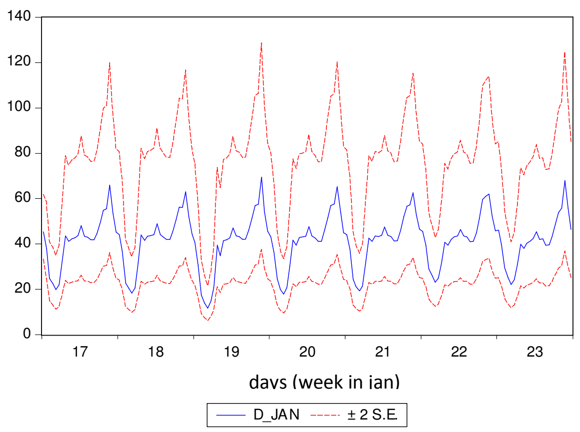
The static forecast is more related to the model adjustment, while the dynamic forecast is more related to the performance of the model prediction for a one-week period. Both adjustment and forecast performances are affected by the volatility in the chosen week. The results are better for the weeks with lower standard deviation, such as the ones analyzed in February, May, July, and August. For the weeks with higher volatility, such as the ones analyzed in January and October, the errors from dynamic forecasting are higher. The model is stable when providing forecasts even if we begin from different observations. For example, Figures 4 and 5 show the

[1] There is not an example for November because it was not possible to choose a week according to the criteria of treated data.

results for the chosen weeks in January and June, respectively. Results for both prediction methods are shown.

Since the dynamic method calculates multi-step forecasts starting from the first period in the forecast sample, compared to the static method, which calculates one-step-ahead forecasts, the uncertainty is higher in the first case, thereby leading to higher standard deviations for each period. By contrast, since static forecasts are calculated based on updated observations for each period, they represent better the data's behavior during the chosen week. The dynamic forecast is the one that produces the information about prediction during one week based on the last observation of the previous week. This forecast is better behaved because it does not consider the updated data.

Dynamic forecast (€/MWh)



Static forecast (€/MWh)

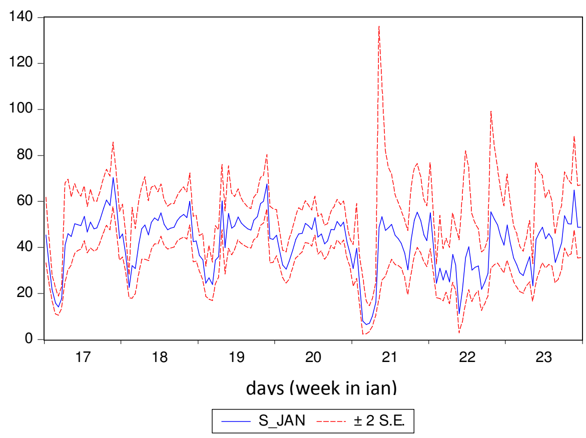


Fig. 4 – Forecasted prices for January week

Dynamic forecast (€/MWh)

Static forecast (€/MWh)

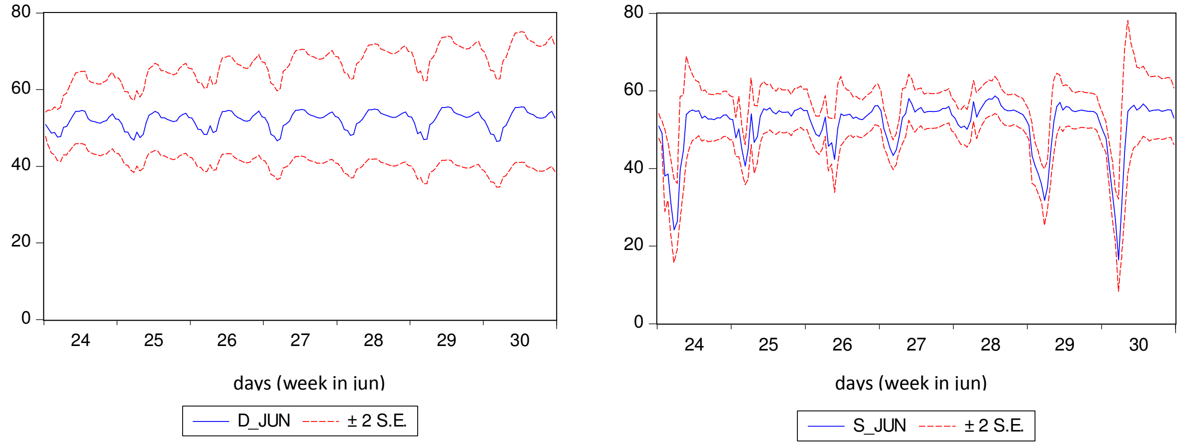


Fig. 5 – Forecasted prices for June week

We also compare some models in terms of forecast performance using simple SARMA or jointly estimating SARMA-GARCH models. We can also compare with the case in which SARMA with lower order is used. Taking the same weeks we analyzed previously, the results are shown in Table 13 in the Appendix. Average results are compared in the graph presented in Figure 6.

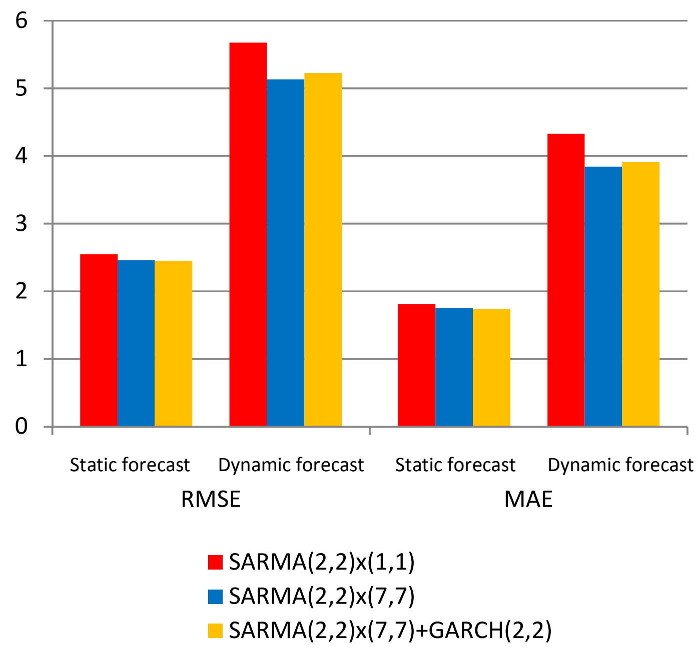


Fig. 6 – Average RMSE and MAE for Spain

Comparing the results above, when we add an additional term of order 168 to capture the weekly seasonality through the SARMA model in hourly data, the forecast results from SARMA(2,2)x(7,7) are improved for all weeks compared to SARMA(2,2)x(1,1). On the other hand, we do not obtain gain of using a combined a SARMA-GARCH when compared to a simple SARMA(2,2)x(7,7) in terms of forecast performance measured through RMSE and MAE. In some cases (and on average for the chosen weeks given the dynamic approach) the simple SARMA model forecast is better than the one using a jointly estimated GARCH model. Even though the SARMA-GARCH better fits to the in-sample data, this is not reflected in the forecast performance to the database with which we are working. This could be related to different causes, such as the in-sample and out-of-sample periods used, or to differences in estimated coefficients for the SARMA parts, which are very similar except for the seasonal moving average term in 168. Nevertheless, the use of the GARCH model is important for understanding the behavior of the volatility.

Instead of working with only one model for the entire hourly series, one alternative is to propose twenty-four different models, one for each hour of the day. In this case, we would work with twenty-four separate time series, and the dynamic forecast methodology for a week would result in seven-steps-ahead forecasts against the 168-steps-ahead we obtained.

5.2. Austria

The same treatment used for outliers in the Spanish data is applied to those in Austrian data. Figures 7(a) and 7(b) present the original price and log-return time series, respectively. Figures 8(a) and 8(b) present the treated price and log return time series, respectively.

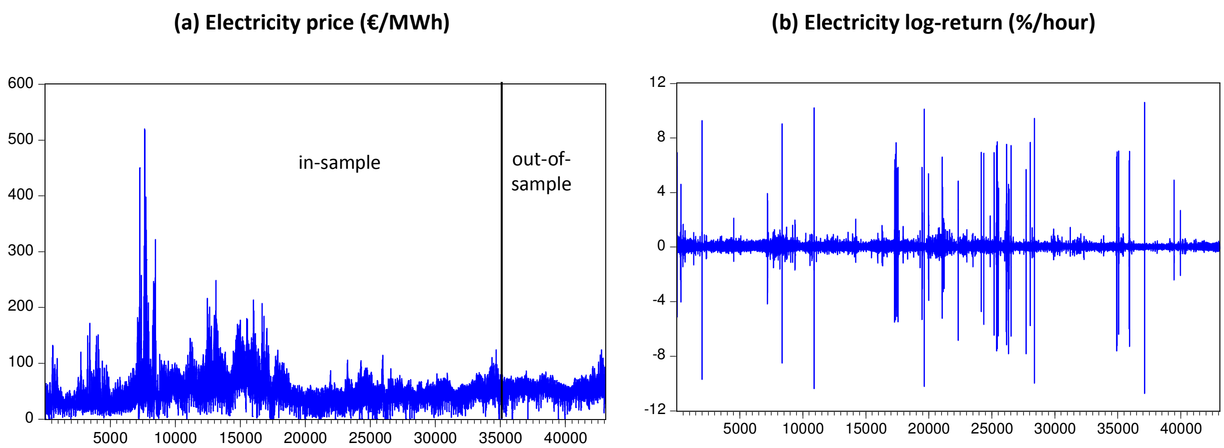


Fig. 7 - Austrian prices and log-return series

(a) Treated electricity price (€/MWh)

(b) Treated electricity log-return (%/hour)

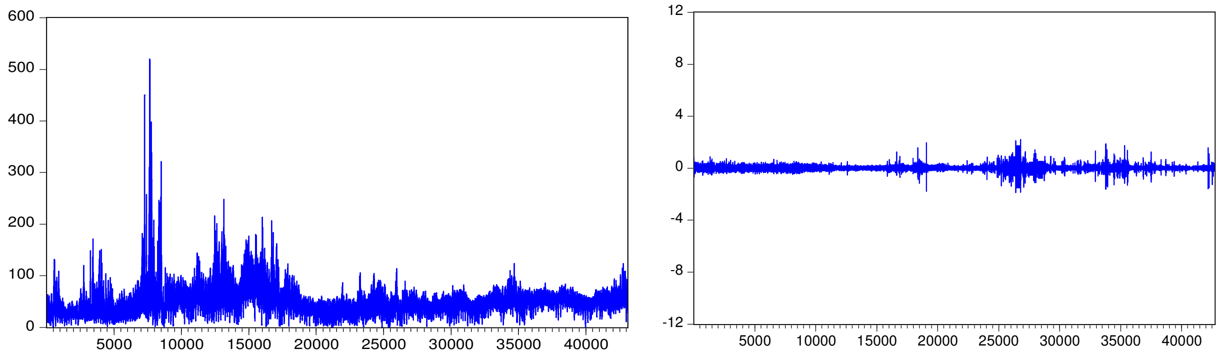


Fig. 8 - Austrian treated prices and log return series

Figure 9(a) presents the sample ACF for the electricity prices and the existence of spikes at lags equal to 24 and multiples revealing an intraday seasonality. A first differentiation and a periodic differentiation of order 24 for log prices are taken again, and Figure 9(b) presents the corresponding autocorrelation function after differentiations. Thus, the series to be modeled is given by:

$$y_t = (1 - L)(1 - L^{24})logP_t \tag{1}$$

(a) ACF for prices **(b) ACF for log prices after differentiations**

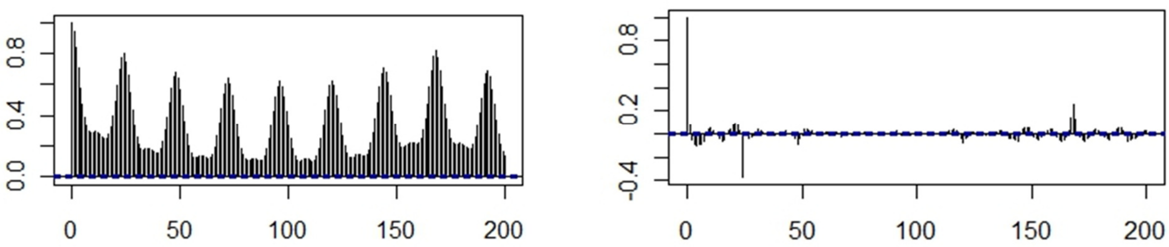


Fig. 9 – ACF for Spanish treated prices and differentiated prices

The parameters of the models are also estimated using an in-sample period data, from January 1, 2007, hour 1, to December 31, 2010, hour 24, as done with Spanish data.

We analyze again simple SARMA models with no conditional volatility treatment. The results are presented in Table 14 in the Appendix. Among the first models tested for both simple and seasonal ARIMA, the SARMA(2,2)x$(1,1)_{24}$ one is a good choice for Austria data as it was for Spanish data. As before, to treat the autocorrelation in lag 168, we test some models adding higher-order terms for the seasonal part. The best-fit model is the SARMA(2,2)x$(7,7)_{24}$ using again terms related only to first and seventh orders for autoregressive and moving average components.

When adding a GARCH model to treat the conditional volatility, the best-fit model for the series is a SARMA(2,2)x$(7,7)_{24}$+GARCH(2,1) using AIC and BIC criteria. The estimation results are presented in Table 8 including the coefficients' values and statistical significance information. Additional information on other models tested is given in Tables 15 and 16 in the Appendix.

Table 8 – Estimated parameters for SARMA-GARCH in entire hourly series

Variable	*Coefficient*	*Std. Error*	*z-Statistic*	*Prob.*
AR(1)	0.810440	0.034670	23.37566	0.0000
AR(2)[2]	-0.001654	0.029230	-0.056590	0.9549
SAR(24)	0.267774	0.001733	154.4779	0.0000
SAR(168)	0.339055	0.000993	341.3417	0.0000
MA(1)	-0.840037	0.034450	-24.38439	0.0000
MA(2)	-0.124447	0.033301	-3.737093	0.0002
SMA(24)	-0.934969	0.000644	-1451.261	0.0000
SMA(168)	-0.061386	0.000655	-93.73916	0.0000
C	0.002155	--	--	--
ARCH(1)	0.394946	0.002542	155.3507	0.0000
ARCH(2)	0.102260	0.004840	21.12821	0.0000
GARCH(1)	0.359625	0.005362	67.06480	0.0000

[2] Even though the AR(2) coefficient is not significant, when we remove it from the model, the results for AIC, BIC, and likelihood function are worse.

For both Spanish and Austrian data, the orders of the best-fit models are very similar, except for an additional term in the GARCH specification. The equations for the model, including SARMA-GARCH terms, are:

$$(1-\varphi_1 L-\varphi_2 L^2)(1-\Phi_{24}L^{24}-\Phi_{168}L^{168})y_t = (1+\theta_1 L+\theta_2 L^2)(1+\Theta_{24}L^{24}+\Theta_{168}L^{168})\epsilon_t \quad (2)$$

$$\sigma_t^2=\alpha_0+\alpha_1\epsilon_{t-1}^2+\alpha_2\epsilon_{t-2}^2+\beta_1\sigma_{t-1}^2 \quad (3)$$

where

φ_1 = 0.810440 θ_1 = -0.840037

φ_2 = -0.001654 θ_2 =-0.124447

Φ_{24} = 0.267774 Θ_{24} =-0.934969

Φ_{168} =0.339055 Θ_{168} =-0.061386

α_0 = 0.002155

α_1 = 0.394946

α_2 =0.102260

β_1 = 0.359625

Analyzing the residuals, they are not normal due to high kurtosis, which is higher than that observed for Spanish data. The histogram for the residuals is presented in Figure 14 in the Appendix. As done with Spanish data, we attempt to model the errors as fat-tailed distributions in the GARCH model, but it is not enough to obtain normally distributed errors. The situation can be attributed to the presence of outliers. Analyzing the ACF for standardized residuals, the same behavior observed for Spanish data is present here for Austrian data, i.e., the seasonality is well captured by the SARMA model.

The forecast performance methodology is the same used before. Table 10 presents the MAE and RMSE measures for one week in each month of the out-of-sample year of 2011, 168 steps-ahead. The weeks are chosen for periods in which there are no treated data. Table 9 also presents the standard deviation for each week, varying from 9.11 to 15.67.

Table 9 – Forecast performance for dynamic and static forecast

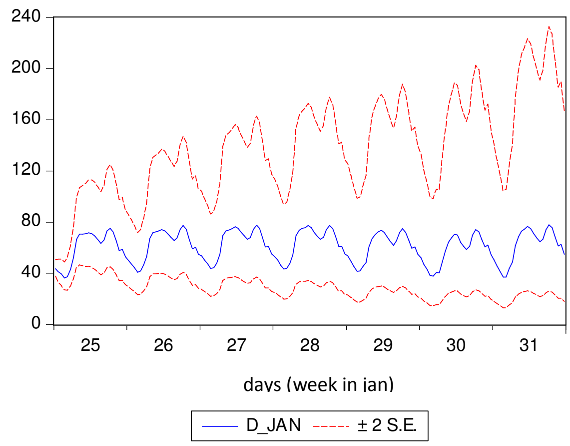
Month	*Week*	*Std. deviation (€/MWh)*	*Static 168 steps-ahead forecast (€/MWh)*		*Dynamic 168 steps-ahead forecast (€/MWh)*	
			RMSE	**MAE**	**RMSE**	**MAE**
January	25/01/2011-31/01/2011	9.11	2.11	1.50	7.74	6.40
February	22/02/2011-28/02/2011	9.93	2.09	1.63	5.30	4.06
March	20/03/2011-26/03/2011	9.30	2.45	1.81	8.06	6.64
April	24/04/2011-30/04/2011	11.52	2.30	1.72	8.17	7.30
May	25/05/2011-31/05/2011	10.05	2.01	1.54	7.61	5.86
June	24/06/2011-30/06/2011	10.05	2.16	1.59	6.38	5.24
July	17/07/2011-23/07/2011	10.56	2.25	1.60	6.18	4.64
August	25/08/2011-31/08/2011	12.42	2.34	1.69	9.40	7.54
September	24/09/2011-30/09/2011	11.61	2.02	1.51	5.40	3.94
October	25/10/2011-31/10/2011	12.13	2.41	1.76	5.13	4.15
November	24/11/2011-30/11/2011	15.67	2.54	1.85	10.76	8.22

Weeks with high standard deviations, such as the one chosen in November, tend to present worse forecast results, but this is not as apparent for Austrian data as it is for Spanish data. Except for this week in November, the analysis in Austria involves weeks that present similar standard deviations, from 9.11 to 12.42, and there is not a direct relation between the standard deviation and the forecast errors.

The volatility is higher for Austrian data than for Spanish data, as observed in the previous analysis. Comparing the results for both countries, the RMSE and MAE measures based on static forecasts are similar on average, representing the adjustment of the models. However, the RMSE and MAE measures for dynamic forecasts are higher on average for Austrian data because of the higher volatility level.

The model is stable when providing forecasts beginning with different observations. Figures 10 and 11 show the results for the chosen weeks in January and June, respectively. Again, both prediction methods are considered. The same observations for Spanish forecast graphs related to dynamic and static forecast can be applied to Austrian data.

Dynamic forecast (€/MWh)



Static forecast (€/MWh)

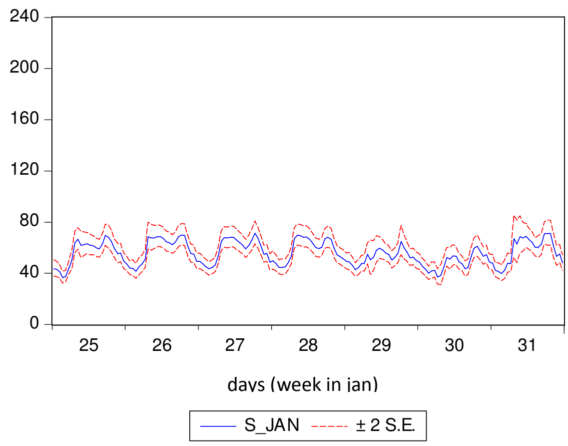


Fig. 10 – Forecasted prices for January week

Dynamic forecast (€/MWh)

Static forecast (€/MWh)

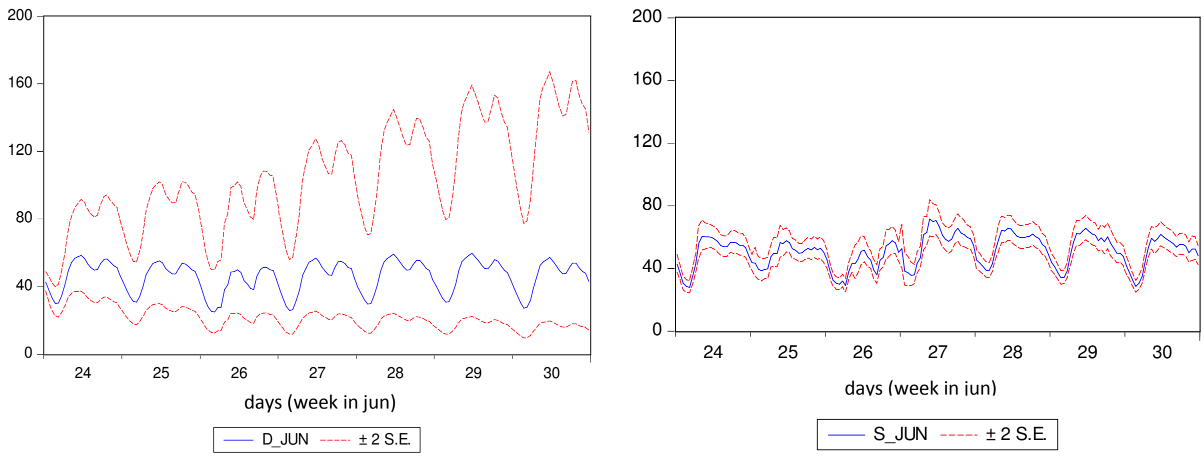


Fig. 11 – Forecasted prices for June week

Taking the best-fit SARMA and SARMA-GARCH models, we can compare them in terms of forecasts for Austrian data as we did for Spanish data. The results are shown in Table 17 in the Appendix. Average results are compared in the graph presented in Figure 12. Again, when we add an additional term of order 168 to capture the weekly seasonality through the SARMA model in hourly data, the forecasted results from SARMA(2,2)x(7,7) present great improvement for almost all the weeks we analyze in comparison to SARMA(2,2)x(1,1). On the other hand, comparing SARMA(2,2)x(7,7) and the corresponding SARMA-GARCH, the simple SARMA model forecast is better than the one using a combined GARCH on average for the chosen weeks, as it happens for Spanish data. Again, this could be related to different causes, such as the periods used as in-sample and out-of-sample, and to differences in estimated coefficients for the SARMA parts, which are higher than observed for Spain, specially comparing the coefficients' values for autoregressive and moving average terms. For Austrian data, which are more volatile than the Spanish ones, the difference in the forecast errors between both models is greater.

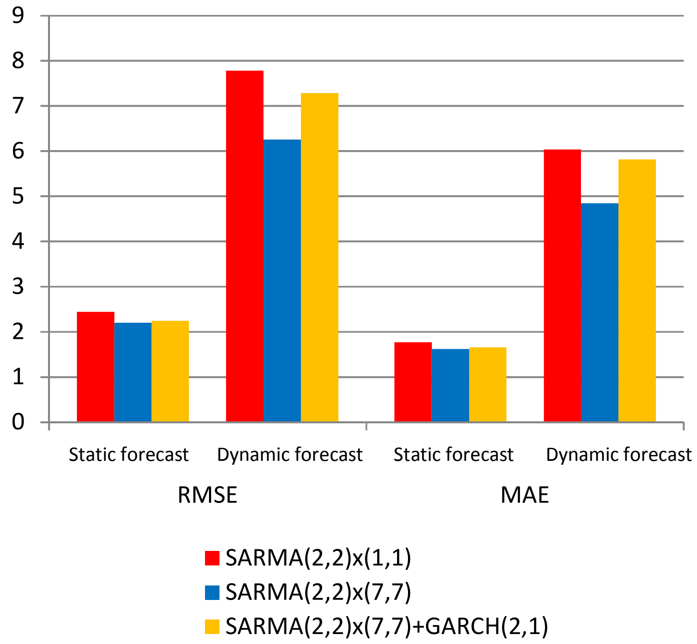


Fig. 12 – Average RMSE and MAE for Austria

6. Conclusions

The ongoing deregulation of electricity industries worldwide is beginning to expose producers, retailers, and consumers alike to uncertain prices. Concomitant pressures to improve energy efficiency as part of global climate change commitments further necessitate the development of better models for decision making under uncertainty. While large power companies may have the expertise to adapt to such a transition, consumers, especially small-scale ones at the building level, will require additional decision support in order to retrofit or to upgrade installed equipment at reasonable cost without exacerbated exposure to risk. Thus, a comparison of methods for analyzing electricity prices is desirable. Subsequently, scenarios based on such analyses could be used as inputs to a customized DSS for risk management.

Following a time-series methodology (Contreras *et al.*, 2003; Garcia *et al.* 2005), we model electricity prices in order to provide a basis for subsequent risk management. We focus on SARMA-GARCH models for hourly time series. The models are fitted for Spanish and Austrian markets, and the results are satisfactory in terms of model adjustment and forecast performance. In both cases, among the SARMA models tested, the SARMA(2,2)x$(1,1)_{24}$ one treats the intraday seasonality, but a better choice to treat the weekly seasonality is the SARMA(2,2)x(7,7) $_{24}$ one. To keep a parsimonious model, the seasonal part presents just the terms related to first and seventh orders for autoregressive and moving average terms. We obtain consistent GARCH models under the coefficient constraints to guarantee wide-sense stationarity for Spanish and Austrian data. Comparing the results, the adjustment of the models for each country is very similar. However, since Austrian data present higher volatility, the forecast performance is slightly better for Spanish data. We also compare the forecast performance for each country using SARMA and SARMA-GARCH models. The results are very similar and, even though the SARMA-GARCH presents better adjustment characteristics, this is not reflected in forecast performances. On average for the analyzed weeks, for both countries, the simple SARMA models present lower forecast errors than the corresponding SARMA-GARCH ones.

Some improvements can be made to the model and will be implemented in future work. First, the ACF shows significant values for lags around 168, representing weekly seasonality. As mentioned, electricity prices present three cycles: annual, weekly, and intraday. In the proposed SARMA model, the SARMA period is related to intraday seasonality, while the weekly seasonality is treated using corresponding lagged terms. One alternative would be to use a double SARMA to treat directly the weekly seasonality. The annual seasonality treatment could also be considered in the SARMA model, which would produce a model with higher orders. Instead of stochastic treatment, another alternative is to model weekly and annual seasonality as a deterministic function as indicated in the literature, thereby leaving the intraday pattern treatment in the SARMA. The log-prices can be decomposed in a deterministic seasonal part, to be modeled using dummies or trigonometric functions, and a stochastic SARMA-GARCH model. Moreover, even using fat-tailed distributions for error terms, the residuals are non-Gaussian. This can be attributed to the presence of outliers, which should be directly modeled alternatively. Dummy variables to control the outlier observations or another alternative to model jumps separately can also be implemented. Finally, considering the forecast performance, an extension of this work would be to compare the results we obtain with the ones from separate models for twenty-four different price series, one for each hour of the day. The model we use is built based on the complete hourly time series, which allows working with only one model for every hour of the day. However, to obtain seven-day period forecasts, for the proposed model, we need 168-steps-ahead forecast against seven-steps-ahead when working with twenty-four separate models. The forecast performance should, thus, be compared.

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8. Appendix

Using Spanish data, some additional models presented the following results:

Table 10 – Alternative models for Spanish data

Model	*AIC*	*BIC*	*LL*
SARMA(1,1)x(1,1)24	-1.72	-1.72	30207.28
SARMA(2,2)x(1,1)24	-1.84	-1.84	32205.8
SARMA(2,2)x(2,2)24	-1.84	-1.84	32208.6
SARMA(3,3)x(1,1)24	-1.84	-1.84	32253.3
SARMA(1,1)x(2,2)24	-1.73	-1.73	30202.3
SARMA(1,1)x(3,3)24	-1.73	-1.73	30185.7
SARMA(1,1)x(4,4)24	-1.75	-1.75	30520.3
SARMA(2,2)x(3,3)24	-1.84	-1.84	32190.6
SARMA(2,2)x(7,7)24*	-1.88	-1.88	32840.8
SARMA(2,2)x(7,7)24 **	-1.88	-1.88	32847.4
SARMA(1,1)x(1,1)24 + GARCH(1,1)	-2.36	-2.36	41386.5
SARMA(2,2)x(1,1)24 + GARCH(1,1)	-2.39	-2.39	41909.3
SARMA(2,2)x(2,2)24 + GARCH(1,1)	-2.39	-2.39	41935.2
SARMA(1,1)x(1,1)24 + GARCH(2,1)	-2.33	-2.33	40796.6
SARMA(2,2)x(1,1)24 + GARCH(2,1)	-2.40	-2.39	42052.1
SARMA(2,2)x(2,2)24 + GARCH(2,1)	-2.43	-2.42	42541.9
SARMA(1,1)x(1,1)24 + GARCH(2,2)	-2.30	-2.30	40355.3
SARMA(2,2)x(1,1)24 + GARCH(2,2)	-2.46	-2.45	43090.4
SARMA(2,2)x(2,2)24 + GARCH(2,2)	-2.46	-2.46	43116.5
SARMA(2,2)x(7,7)24* + GARCH(1,1)	-2.43	-2.43	42450.3
SARMA(2,2)x(7,7)24* + GARCH(2,1)	-2.50	-2.49	43607.4
SARMA(2,2)x(7,7)24* + GARCH(2,2)	-2.52	-2.52	43977.5

*Seasonal terms for AR(24), AR(168), MA(24), MA(168)
** Seasonal terms for AR(24), AR(48), AR(168), MA(24), MA(48) MA(168)

Fitting only a SARMA model before considering the GARCH part, the best-fit models, are given by SARMA(2,2)x$(1,1)_{24}$ considering AIC and BIC and keeping parsimonious models, when the weekly seasonality is not included, and by SARMA(2,2)x$(7,7)_{24}$, to treat the ACF observed in lag 168 related to weekly seasonality, reminding that in the latter case only terms of first and seventh orders are used in the seasonal part. When including the GARCH model, the best-fit model was the SARMA(2,2)x$(7,7)_{24}$+GARCH(2,2). Figure 13 presents the histogram for standardized residuals (skewness of -0.39 and kurtosis of 10.96).

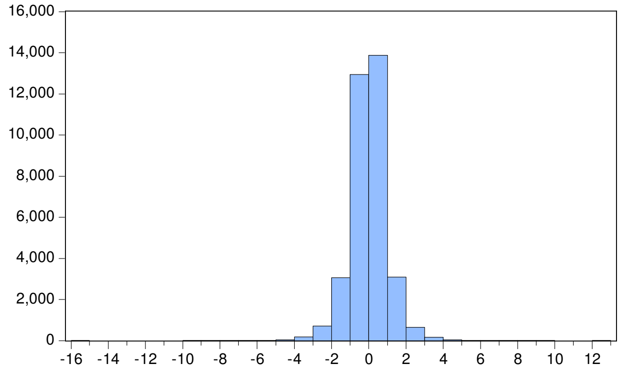

Fig. 13 – Histogram for standardized residuals (Spain)

Besides the estimated parameters for SARMA(2,2)x(7,7) $_{24}$-GARCH(2,2), which are shown in the text, the ones for SARMA(2,2)x(1,1)$_{24}$ and SARMA(2,2)x(7,7)$_{24}$ are given in the tables below.

Table 11 – Estimated parameters for SARMA (2,2)x(1,1)$_{24}$ – Spanish data

Variable	*Coefficient*	*Std. Error*	*t-Statistic*	*Prob.*
AR(1)	1.208732	0.023585	51.24916	0.0000
AR(2)	-0.368038	0.019728	-18.65519	0.0000
SAR(24)	0.216567	0.006921	31.29316	0.0000
MA(1)	-1.207390	0.024984	-48.32602	0.0000
MA(2)	0.207581	0.024979	8.310117	0.0000
SMA(24)	-0.834499	0.003912	-213.3055	0.0000

Table 12 – Estimated parameters for SARMA (2,2)x(7,7)$_{24}$ – Spanish data

Variable	*Coefficient*	*Std. Error*	*t-Statistic*	*Prob.*
AR(1)	1.206598	0.026278	45.91670	0.0000
AR(2)	-0.365502	0.021777	-16.78396	0.0000
SAR(24)	0.209610	0.006368	32.91528	0.0000
SAR(168)	0.271118	0.005989	45.27073	0.0000
MA(1)	-1.228789	0.027703	-44.35511	0.0000
MA(2)	0.230997	0.027650	8.354381	0.0000
SMA(24)	-0.839896	0.003809	-220.5284	0.0000
SMA(168)	-0.130293	0.003730	-34.93549	0.0000

The forecast performance of the three models are presented in Table 13.

Table 13 – Forecast performance for SARMA and SARMA-GARCH models

Month	***Week***	***SARMA(2,2)x(1,1)***				***SARMA(2,2)x(7,7)***				***SARMA(2,2)x(7,7)+ GARCH(2,2)***			
		Static forecast		***Dynamic forecast***		***Static forecast***		***Dynamic forecast***		***Static forecast***		***Dynamic forecast***	
		RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
January	17/01/2011 - 23/01/2011	5.44	3.77	9.37	7.78	5.32	3.70	8.88	6.88	5.46	3.80	9.39	7.60
February	01/02/2011 - 07/02/2011	2.00	1.53	5.11	3.95	1.97	1.51	3.99	2.86	1.93	1.45	3.68	2.72
March	20/03/2011 - 26/03/2011	2.31	1.72	4.57	3.45	2.25	1.67	4.24	3.42	2.20	1.63	3.82	3.14
April	24/04/2011 - 30/04/2011	2.67	1.80	6.09	4.57	2.62	1.81	5.89	4.33	2.71	1.87	6.21	4.57
May	25/05/2011 - 31/05/2011	1.51	1.09	3.50	2.51	1.45	1.08	3.43	2.6	1.32	0.97	3.39	2.37
June	24/06/2011 - 30/06/2011	2.81	1.70	6.34	3.82	2.72	1.63	5.66	3.27	2.74	1.60	6.23	3.45
July	25/07/2011 - 31/07/2011	2.03	1.38	4.06	3.15	1.93	1.32	3.58	2.77	1.94	1.33	3.78	3.07
August	25/08/2011 - 31/08/2011	1.56	1.20	2.89	2.44	1.51	1.17	2.58	2.18	1.48	1.14	2.55	2.16
September	24/09/2011 - 30/09/2011	2.22	1.66	5.49	4.61	2.09	1.54	4.63	3.92	2.09	1.53	4.74	3.92
October	17/10/2011 - 23/10/2011	2.92	2.26	9.35	7.01	2.73	2.09	8.43	6.15	2.63	2.04	8.46	6.10
Average		2.55	1.81	5.68	4.33	2.46	1.75	5.13	3.84	2.45	1.74	5.23	3.91

Using Austrian data, some additional models presented the following results:

Table 14 – Alternative models for Austrian data

Model	***AIC***	***BIC***	***LL***
SARMA(1,1)x(1,1)24	-1.24	-1.24	21751.4
SARMA(2,2)x(1,1)24	-1.30	-1.30	22793.7
SARMA(2,2)x(2,2)24	-1.30	-1.30	22799.2
SARMA(3,3)x(1,1)24	-1.30	-1.30	22808.9
SARMA(1,1)x(2,2)24	-1.24	-1.24	21757
SARMA(1,1)x(3,3)24	-1.24	-1.24	21758.4
SARMA(1,1)x(4,4)24	-1.26	-1.26	22030.5
SARMA(2,2)x(3,3)24	-1.30	-1.30	22803.2
SARMA(2,2)x(7,7)24*	-1.36	-1.36	23821.4

SARMA(2,2)x(7,7)24**	-1.37	-1.36	23848.8
SARMA(1,1)x(1,1)24 + GARCH(1,1)	-1.87	-1.87	32812
SARMA(2,2)x(1,1)24 + GARCH(1,1)	-1.88	-1.88	33031.4
SARMA(2,2)x(2,2)24 + GARCH(1,1)	-1.89	-1.89	33044.4
SARMA(2,2)x(7,7)24* + GARCH(1,1)	-1.88	-1.88	32792.2
SARMA(1,1)x(1,1)24 + GARCH(2,1)	-1.87	-1.87	32819.1
SARMA(2,2)x(1,1)24 + GARCH(2,1)	-1.70	-1.70	29823.1
SARMA(2,2)x(2,2)24 + GARCH(2,1)	-1.89	-1.89	33087.8
SARMA(2,2)x(7,7)24* + GARCH(2,1)	-1.95	-1.95	34060.1
SARMA(2,2)x(7,7)24*+ GARCH(2,1)	-1.34	-1.34	23350.2
SARMA(1,1)x(1,1)24 + GARCH(2,2)	-1.81	-1.81	31705.1
SARMA(2,2)x(1,1)24 + GARCH(2,2)	-1.34	-1.34	23448.9
SARMA(2,2)x(2,2)24 + GARCH(2,2)	-1.89	-1.89	33145.1
SARMA(2,2)x(7,7)24* + GARCH(2,2)	-1.89	-1.88	32902.3
SARMA(2,2)x(7,7)24* + GARCH(2,2)	-1.89	-1.88	33002.9

*Seasonal terms for AR(24), AR(168), MA(24), MA(168)
** Seasonal terms for AR(24), AR(48), AR(168), MA(24), MA(48) MA(168)

Again, as observed for Spanish data, fitting only a SARMA model before considering the GARCH part, the best-fit model is given by SARMA(2,2)x$(1,1)_{24}$, when the weekly seasonality is not included, and by SARMA(2,2)x$(7,7)_{24}$, to treat the ACF observed in lag 168 related to weekly seasonality, reminding that in the latter case only terms of first and seventh orders are used in the seasonal part. When including the GARCH model, the model SARMA(2,2)X$(2,2)_{24}$+GARCH(2,1) was the best-fit one. Figure 14 presents the histogram for standardized residuals (skewness of -1.37 and kurtosis of 33.73).

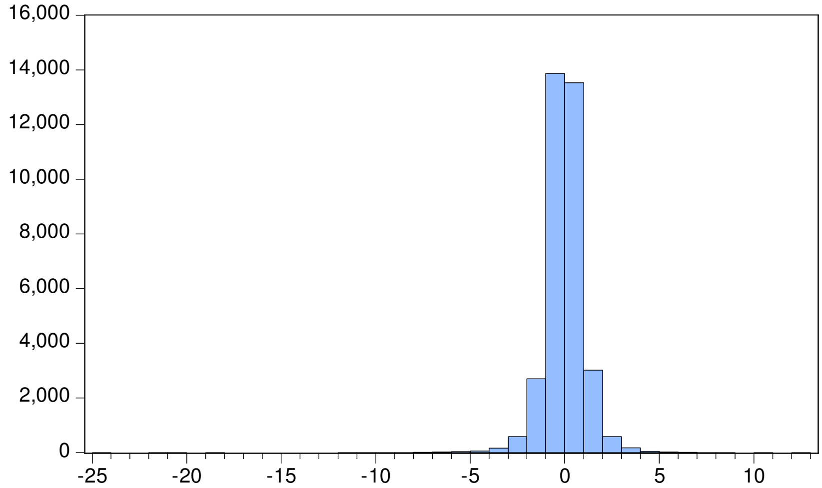

Fig. 14 – Histogram for standardized residuals (Austria)

The estimated parameters for SARMA(2,2)x(1,1)$_{24}$ and SARMA(2,2)x(7,7)$_{24}$ are given in the tables below.

Table 15 – Estimated parameters for SARMA (2,2)x(1,1)$_{24}$ – Austrian data

Variable	*Coefficient*	*Std. Error*	*t-Statistic*	*Prob.*
AR(1)	1.106258	0.007145	154.8195	0.0000
AR(2)	-0.206689	0.006855	-30.14980	0.0000
SAR(24)	0.244788	0.005856	41.80421	0.0000
MA(1)	-1.051754	0.005035	-208.8825	0.0000
MA(2)	0.052033	0.005035	10.33404	0.0000
SMA(24)	-0.931963	0.002174	-428.7369	0.0000

Table 16 – Estimated parameters for SARMA (2,2)x(7,7)$_{24}$ – Austrian data

Variable	*Coefficient*	*Std. Error*	*t-Statistic*	*Prob.*
AR(1)	1.087901	0.044008	24.72068	0.0000
AR(2)	-0.201137	0.039143	-5.138455	0.0000
SAR(24)	0.247345	0.005805	42.60579	0.0000
SAR(168)	0.307716	0.005196	59.22002	0.0000
MA(1)	-1.092958	0.044754	-24.42130	0.0000
MA(2)	0.094461	0.044699	2.113255	0.0346
SMA(24)	-0.932932	0.002951	-316.1383	0.0000
SMA(168)	-0.060217	0.002897	-20.78614	0.0000

The forecast performance of the three models are presented in Table 17.

Table 17 – Forecast performance for SARMA and SARMA-GARCH models

Month	***Week***	***SARMA(2,2)x(1,1)***				***SARMA(2,2)x(7,7)***				***SARMA(2,2)x(7,7)+ GARCH(2,1)***			
		Static forecast		***Dynamic forecast***		***Static forecast***		***Dynamic forecast***		***Static forecast***		***Dynamic forecast***	
		RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
January	25/01/2011-31/01/2011	2.78	2.10	9.08	7.57	2.04	1.45	4.50	3.57	2.11	1.50	7.74	6.40
February	22/02/2011-28/02/2011	2.45	1.84	7.05	6.22	2.07	1.61	5.33	4.45	2.09	1.63	5.30	4.06
March	20/03/2011-26/03/2011	2.47	1.74	5.88	4.35	2.41	1.76	6.24	5.19	2.45	1.81	8.06	6.64
April	24/04/2011-30/04/2011	2.31	1.73	9.87	7.46	2.22	1.66	8.75	6.70	2.30	1.72	8.17	7.30
May	25/05/2011-31/05/2011	2.00	1.46	7.35	5.34	1.93	1.45	6.90	5.23	2.01	1.54	7.61	5.86
June	24/06/2011-30/06/2011	2.47	1.70	5.75	4.27	2.12	1.55	3.98	3.16	2.16	1.59	6.38	5.24
July	17/07/2011-23/07/2011	2.33	1.62	7.54	5.53	2.20	1.57	6.55	4.78	2.25	1.60	6.18	4.64
August	25/08/2011-31/08/2011	2.48	1.80	9.46	7.97	2.32	1.67	7.20	5.74	2.34	1.69	9.40	7.54
September	24/09/2011-31/09/2011	2.18	1.57	6.90	5.45	1.99	1.49	5.51	4.17	2.02	1.51	5.40	3.94
October	25/10/2011-31/10/2011	2.68	1.98	6.55	5.30	2.43	1.78	4.90	4.00	2.41	1.76	5.13	4.15
November	24/11/2011-30/11/2011	2.66	1.90	10.16	6.97	2.43	1.80	8.96	6.33	2.54	1.85	10.76	8.22
Average		2.44	1.77	7.78	6.04	2.20	1.62	6.26	4.85	2.24	1.66	7.29	5.82