Updated Symbolic Model Specification

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1 Symbolic Model Specification

This document contains the SMS implemented in the EnRiMa "Stochastic optimisation" prototype. Thus, it is an update of D4.2. Deliverable D4.3 contains an explanation and justification of the changes, mainly due to the "by-node" formulation finally adopted. This document contains:

1. The mathematical representation of the entities in the model:
    - Sets, including standard sets, aliases, subsets, and conditional sets;
    - Constants (scalars, in GAMS language);
    - Decision variables (DV);
    - Parameters;
    - Equations, classified into operational and strategic constraints, and objective function.
2. The data representation of the SMS within the Solver Manager.

1.1 Scenario Tree Representation

This subsection contains an explanation of the symbols used for the scenario tree as well as the relationship between the tree nodes and other entities in the model. Scenario trees are widely used in stochastic programming to discretize the huge, usually infinite, number of possible outcomes of the random variables in a stochastic model. Thus, a scenario tree gathers the most probable scenarios resulting from a combination of all random variables (stochastic parameters using the SMS language). Several size-reduction techniques can be used in order to make the problems computationally tractable. As a result, typical scenario trees are implicitly represented within the whole SMS. For the sake of clarity, we briefly explain here the scenario trees used in the models.

A scenario tree can be represented graphically as an acyclic graph consisting of nodes and arcs, where the root node has no parent (predecessor node), each node may have one or more children, and each node can only have one parent. The number of terminal nodes (leaves), which do not have children, determines the number of scenarios considered. Each scenario is a path from the root node to a leaf node. The nodes represent states of the system at a particular time, where decisions are made. The root node corresponds to the beginning of the planning horizon. Arcs represent the precedence relationship between nodes with an associated probability of occurrence. Therefore, in addition to the node identifier (v in the SMS), the following information is required:

- The parent node of each node. It is represented by the conditional set $\mathcal{V}_{Pa}^{v}$.
- The probability of each node. It is represented by the parameter PR^{v}.
- The time period of each node. It is represented by the parameter PT^{v}.

In addition, the conditional set $\mathcal{A}_{Ages}^{i,v}$ is defined as the possible technologies' ages at a given node for a given technology. For example, if period of node 3 is equal to 3, and at the beginning of the planning horizon a unit of CHP technology whose age was 5 years was already in the building, then the possible

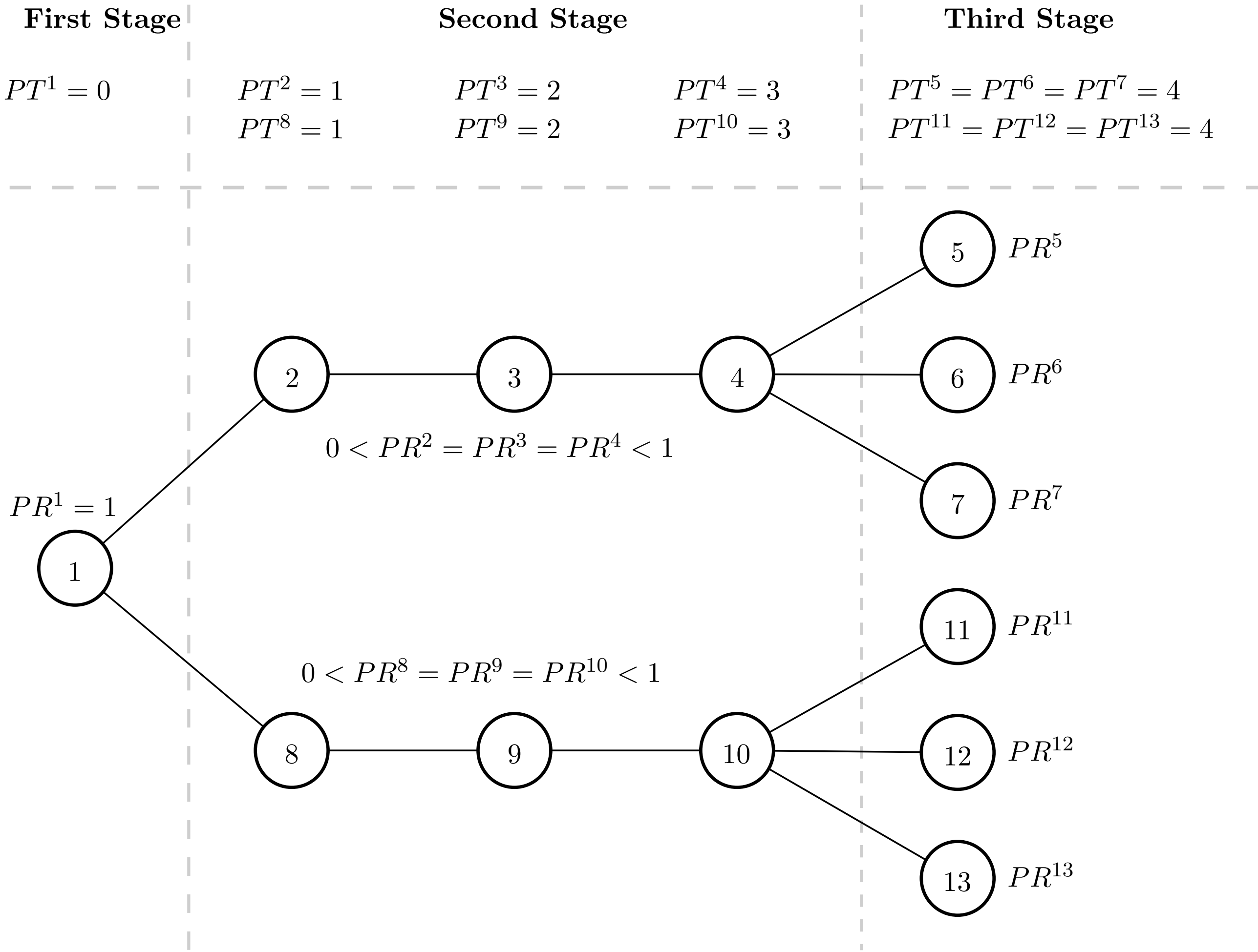

Figure 1: Scenario tree symbolic representation.

ages for that technology at node 3 are {0,1,2,3,8}.

Figure 1 shows a simple scenario tree with all the symbols and expressions used in the SMS. Circles represent nodes with the node index (v) displayed inside. Nodes in the same column correspond to the same time period (PT^v). Each node is linked to its parent through the conditional set $\mathcal{V}_{Pa}^{v}$ and has a probability (PR^v) associated to its parent's branching. The represented tree corresponds to a three-stage stochastic problem, where new information arrives at periods 1 and 4.

2 Model Entities

2.1 Sets

The information presented for each set is:

- Symbol. Small latin letter for sets and aliases, see below for subsets and conditional sets.
- Short description.
- Domain, representing the set by calligraphic capital letters.
- Tag, which is a label used for interfacing purposes, in square brackets.
- Long description.

a Technology age, $a \in \mathcal{A}$ [age]. This set is used to model the effect of aging on the capacity and the costs of the technologies.

i Energy technology, $i \in \mathcal{I}$ [enerTech]. Equipment available in the building, or suitable to be installed. This equipment can be: (1) Energy generator, (2) Energy storage, or (3) Energy saver. Each element of the set is a specific model of a type of technology (e.g., CHP), with different features.

k Energy type, $k \in \mathcal{K}$ [enerType]. Type of energy that will be used in the building.

l Type of pollutant, $l \in \mathcal{L}$ [pollutant]. Energy generation and consumption generate emissions to the environment. The amount of a building's emissions of each pollutant depends on the emission ratios. The total emissions can be constrained by policy makers. Their minimization can also be an objective for certain decision makers.

m Operational profile, $m \in \mathcal{M}$ [profile]. This set gathers the representative profiles considered in the model to link the short- and long-term performance of the energy systems in the building: this short-term performance is scaled to the long term through a weight factor given as a parameter value.

n Energy tariff, $n \in \mathcal{N}$ [tariff]. This set contains the tariffs available throughout the decision time span. It is possible that not all the tariffs are available at each scenario tree node.

t Short-term period, $t \in \mathcal{T}$ [time]. These are the periods when operational decisions are made. Such decisions concern how much energy of each type must flow through the building energy systems, from markets to demand.

v Tree node, $v \in \mathcal{V}$ [node]. This set contains the nodes in the scenario tree. For each node, its time period (cf., p index in the deterministic model), probability, and parent node must be specified

2.2 Aliases

k' Output energy type, $k' \in \mathcal{K}$ [outEner]. This index is used in order to distinguish the type of the energy input and output when using generators.

v' Parent nodes, $v' \in \mathcal{V}$ [parentNode]. This index is used to map the parent node of each node.

2.3 Subsets

Subsets are represented through the symbol of the set in which they are contained in calligraphic font and the symbol of the subset as subscript in *italics*.

$\mathcal{A}_{New}$ Age = 0, $\mathcal{A}_{New} \subset \mathcal{A}$ [ageNew]. This set contains only the element 0 from the age set.

$\mathcal{A}_{Old}$ Age != 0, $\mathcal{A}_{Old} \subset \mathcal{A}$ [ageOld]. This set contains all the elements from the age set except 0.

$\mathcal{I}_{Cn}$ Continuously-sized technologies, $\mathcal{I}_{Cn} \subset \mathcal{I}$ [contTech]. Technologies are continuously sized if they do not have a nominal capacity and the investment can be done by power units.

$\mathcal{I}_{Ds}$ Discretely-sized technologies, $\mathcal{I}_{Ds} \subset \mathcal{I}$ [discTech]. Technologies are discretely sized if they have a nominal capacity and the investment has to be done by devices.

$\mathcal{I}_{Gen}$ Energy-generation technologies, $\mathcal{I}_{Gen} \subset \mathcal{I}$ [genTech]. Technologies that receive energy as input and return other type(s) of energy as output.

$\mathcal{I}_{PU}$ Passive technologies (unitary), $\mathcal{I}_{PU} \subset \mathcal{I}$ [passiveTech]. Passive technologies which have a multiplicative effect on the demand, that is, the higher the demand, the higher the savings. They entail savings over the use of the energy regardless of the building dimensions.

$\mathcal{I}_{Sto}$ Storage technologies, $\mathcal{I}_{Sto} \subset \mathcal{I}$ `[stoTech]`. Devices that store a type of energy from the market or the generation technologies and can release this energy to meet the demand. These technologies are subject to losses both at the input and at the output. The storage is also uncharged at a given ratio even if energy is not released to be consumed.

$\mathcal{K}_{Cool}$ Type of energy for cooling, $\mathcal{K}_{Cool} \subset \mathcal{K}$ `[coolEner]`.

$\mathcal{K}_{Dem}$ Types of energy on the demand side, $\mathcal{K}_{Dem} \subset \mathcal{K}$ `[demEner]`. This is the union of the subsets Elec, Heat and Cool.

$\mathcal{K}_{Elec}$ Type of energy for electricity, $\mathcal{K}_{Elec} \subset \mathcal{K}$ `[elecEner]`.

$\mathcal{K}_{EP}$ Energy to purchase, $\mathcal{K}_{EP} \subset \mathcal{K}$ `[enerPurchase]`. Types of energy which can be purchased.

$\mathcal{K}_{ES}$ Energy to be sold, $\mathcal{K}_{ES} \subset \mathcal{K}$ `[enerSale]`. Types of energy which can be sold.

$\mathcal{K}_{Heat}$ Type of energy for heat, $\mathcal{K}_{Heat} \subset \mathcal{K}$ `[heatEner]`.

$\mathcal{K}_{Ren}$ Renewable energy, $\mathcal{K}_{Ren} \subset \mathcal{K}$ `[enerRen]`. Renewable energy does not need energy balance constraints.

$\mathcal{N}_{TP}$ Purchasing tariffs, $\mathcal{N}_{TP} \subset \mathcal{N}$ `[purchTariffs]`. This subset contains the tariffs available to buy energy.

$\mathcal{N}_{TS}$ Sales tariffs, $\mathcal{N}_{TS} \subset \mathcal{N}$ `[salesTariffs]`. This subset contains the tariffs available to sell energy.

$\mathcal{V}_{Fut}$ Future nodes, $\mathcal{V}_{Fut} \subset \mathcal{V}$ `[futureNodes]`. All the nodes that are not the root node.

$\mathcal{V}_{Root}$ Root node, $\mathcal{V}_{Root} \subset \mathcal{V}$ `[rootNode]`. This subset only contains the root node and is needed to identify states at time 0, for example, existing technologies.

2.4 Conditional Sets

Conditional (or multidimensional) sets are represented as the symbol of the main set in calligraphic font, the symbol of the conditional set as subscript in *italics* font, and the index of the 'input' set(s) as superscript.

$\mathcal{A}^{i,v}_{Ages}$ Possible ages of a technology at a node, $i \in \mathcal{I}, v \in \mathcal{V}$ `[agesNode]`. This conditional set provides all the possible ages that technologies may have at a given node.

$\mathcal{K}^{i}_{In}$ Input energy types for a technology, $i \in \mathcal{I}_{Gen}$ `[inputEner]`. Generation technologies can utilize different types of energy to generate the output.

$\mathcal{K}^{i}_{Out}$ Output energy types for a technology, $i \in \mathcal{I}_{Gen}$ `[outEnergy]`. Generation technologies provide one or more output energy types.

$\mathcal{K}^{i}_{Po}$ Principal energy of technologies, $i \in \mathcal{I}$ `[principalEner]`. Each generation technology has a principal output type of energy (when more than one). For storage technologies, the input and output types of energy are the same. For passive measures, it is the type of energy which is saved.

$\mathcal{N}^{k}_{Pur}$ Purchase tariffs for each energy type, $k \in \mathcal{K}$ `[enerBtariff]`. Conditional set to make purchase tariffs for energy types available.

$\mathcal{N}^{k}_{S}$ Sales tariffs for each energy type, $k \in \mathcal{K}$ `[enerStariff]`. Conditional set to make sales tariffs for energy types available.

$\mathcal{N}^{k}_{Tr}$ Energy that can be traded in each market, $k \in \mathcal{K}$ `[enerTrade]`. This conditional set is the union of $\mathcal{N}^{k}_{Pur}$ and $\mathcal{N}^{k}_{S}$.

$\mathcal{T}^{m}_{Tm}$ Short-term periods by profile, $m \in \mathcal{M}$ `[perProfile]`. Each profile can contain several operational periods, whose duration is modeled through the DM parameter.

$\mathcal{V}^{v}_{Pa}$ Parent for each node, $v \in \mathcal{V}$ [parentNodes]. This conditional set contains the relationship between each node and its parent. Note that the parent node is represented as $Pa(v)$ when it is used as an index in an expression.

2.5 Constants

DR Discount rate, per year[dRate].

2.6 Parameters

For each parameter (applies also to variables), the following information is shown:

- Expression, formed by the parameter/variable symbol and the sets' indices that apply;
- Short description;
- Measurement units;
- Nature (deterministic/stochastic);
- Tag and datatype, in square brackets and separated by ":", for interfacing purposes;
- Domain for the indices of the expression;
- Long description;

$AF_i^{v,m,t}$ Availability factor for a technology (kWh/kWh, stochastic, [genAvail:decimal]). $i \in \mathcal{I}$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}^m_{Tm}$. The capacity of a technology may change throughout the optimization horizon. For example, photovoltaic panels do not have the same performance during the day and they even do not work during the night. The factor can also be used to model the availability of future technologies.

AG_i^a Technology aging factor (kW/kWh, deterministic, [techAging:decimal]). $i \in \mathcal{I}$, $a \in \mathcal{A}$. This parameter adjusts the total capacity of a technology throughout its lifetime. The superindex is for the age. That is, at age 0, a given technology (e.g., CHP Dachs 5.5) has factor 1, which reduces at some rate each year.

$B_{k,n}$ Primary energy needed to produce final-use energy (kWh/kWh, stochastic, [marketPrimEff:decimal]). $k \in \mathcal{K}$, $n \in \mathcal{N}^k_{Pur}$. Units of primary energy required to produce one unit of a type of energy available from a market where processed energy can be bought.

$CD_i^{v,a}$ Technology decommissioning cost (EUR/kW, stochastic, [techDecomCost:decimal]). $i \in \mathcal{I}$, $v \in \mathcal{V}$, $a \in \mathcal{A}$. Decommissioning a technology may lead to a removal cost or revenue from selling the equipment (in such a case, the value of the parameter is negative). It can be related to the installation cost.

CI_i^v Technology installation cost (EUR/kW, stochastic, [techInstCost:decimal]). $i \in \mathcal{I}$, $v \in \mathcal{V}$.

$CM_i^{v,a}$ Technology maintenance cost (EUR/kW, EUR/kWh, stochastic, [techMaintCost:decimal]). $i \in \mathcal{I}$, $v \in \mathcal{V}$, $a \in \mathcal{A}$. This is a fixed cost per capacity installed. It may be linked to the installation cost.

$CO^{v}_{i,k}$ Technology operation cost (EUR/kWh, stochastic, [genOperCost:decimal]). $i \in \mathcal{I}_{Gen}$, $k \in \mathcal{K}^{i}_{Out}$, $v \in \mathcal{V}$. This parameter is used when the supplier/maintainer's tariff is quoted per operated 'kWh'.

$D^{v,m,t}_{k}$ Energy demand (kWh, stochastic, [enerDemand:decimal]). $k \in \mathcal{K}_{Dem}$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}^{m}_{Tm}$. Total energy load of the building for a type of energy, during each short-term (operational) period.

DM^{m} Weight (scaling factor) for the operational profile in the objective (days, deterministic, [profileWeight:integer]). $m \in \mathcal{M}$. This parameter is used to scale the operational system performance (energy, cost) to the strategic time resolution.

DT^{m} Duration of the short-term period within a given profile. (hours, deterministic, [durOper:decimal]). $m \in \mathcal{M}$. The sum over the durations of all the operational periods must correspond to a whole day. This parameter is used to convert energy to power or vice versa.

$EC^{v}_{i,k,k'}$ Output energy generated from one unit of input energy (kWh/kWh, stochastic, [genConvFact:decimal]). $i \in \mathcal{I}_{Gen}$, $k \in \mathcal{K}^{i}_{In}$, $k' \in \mathcal{K}^{i}_{Out}$, $v \in \mathcal{V}$. This is a conversion factor. It is applied to the input energy of a technology to compute the output energy of this technology. Both types of energy can be the same or different. We may also have several types of output and input energy (e.g., gas, biogas)

EF^{v} Required building energy efficiency (unitless, stochastic, [buildingEff:decimal]). $v \in \mathcal{V}$.

G_{i} Technology capacity (kW/Device, deterministic, [techCapacity:decimal]). $i \in \mathcal{I}$. Nominal capacity of each device of a given technology. For continuous technologies, its value is 1.

IL^{v} Investment limit (EUR, stochastic, [invLimit:integer]). $v \in \mathcal{V}$. This is needed when the building has a budget limit for investing in technologies.

$LC^{v}_{k,l,n}$ Pollution emissions by energy purchases (kg/kWh, stochastic, [marketEmission:decimal]). $k \in \mathcal{K}$, $l \in \mathcal{L}$, $n \in \mathcal{N}^{k}_{Pur}$, $v \in \mathcal{V}$. Mean rate of emission of a pollutant from processed energy purchased in the market.

$LH^{v}_{i,k,l}$ Pollution emissions by generating technologies (kg/kWh, stochastic, [genEmission:decimal]). $i \in \mathcal{I}_{Gen}$, $k \in \mathcal{K}^{i}_{In}$, $l \in \mathcal{L}$, $v \in \mathcal{V}$. Amount of pollutant that is emitted by a generation technology during its operation, for each type of input energy.

LP^{v}_{i} Physical Limit (Devices/KW/kWh, stochastic, [techPhysLim:decimal/integer]). $i \in \mathcal{I}$, $v \in \mathcal{V}$. Number of units or capacity of a technology that can be installed at the site at a time.

$ME_{k,n}$ Maximum purchase/sale of a type of energy under a given contract (kW, deterministic, [maxTrading:decimal]). $k \in \mathcal{K}$, $n \in \mathcal{N}^{k}_{Tr}$.

$OA_{i,k}^v$ Fraction of storage lower limit (kWh/kWh, stochastic, [stoLowerLim:decimal(0..1)]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$. Minimum fraction of the capacity that must be charged in an energy-storage technology.

$OB_{i,k}^v$ Fraction of storage upper limit (kWh/kWh, stochastic, [stoUpperLim:decimal(0..1)]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$. Maximum fraction of the capacity that must be charged in an energy-storage technology.

$OD_{i,k}^v$ Energy demand reduction for a passive technology (kWh/kWh, stochastic, [pasDemandRed:decimal]). $i \in \mathcal{I}_{PU}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$. For each unit of a passive technology, the total demand is reduced by some value.

$OI_{i,k}^v$ Charging ratio to storage (kWh/kWh, stochastic, [stoChargRatio:decimal]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$. Units of energy available for each unit sent to energy-storage technology.

$OO_{i,k}^v$ Discharging ratio from storage (kWh/kWh, stochastic, [stoDisChargRatio:decimal]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$. Units of energy needed to be discharged from storage in order to deliver one unit of energy to the demand.

$OS_{i,k}$ Energy storage availability (kWh/kWh, deterministic, [stoAvail:decimal(0..1)]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$. This parameter models the energy loss of a storage technology over the time. It represents the units of energy available for each unit of energy stored after each operational period.

$OX_{i,k}^v$ Maximum discharge rate (kW/kWh, stochastic, [stoDisRate:decimal]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$. Maximum energy discharge rate per unit of storage capacity.

$OY_{i,k}^v$ Maximum charge rate (kW/kWh, stochastic, [stoChargeRate:decimal]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$. Maximum energy charge rate per unit of storage capacity.

PL_l^v Pollution limit (kg, stochastic, [emissLimit:integer]). $l \in \mathcal{L}$, $v \in \mathcal{V}$. This is a constraint for the building for each year.

$PP_{k,n}^{v,m,t}$ Energy purchasing cost (EUR/kWh, stochastic, [marketEnerPurCost:decimal]). $k \in \mathcal{K}$, $n \in \mathcal{N}_{Pur}^k$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. This is the cost of the energy in markets where it can be bought. If there is no ToU tariff, the cost is equal for all operational periods within the same strategic period.

PR^v Probability of the node (unitless, stochastic, [nodeProb:decimal]). $v \in \mathcal{V}$.

PT^v Time period of the node (unitless, stochastic, [nodePeriod:decimal]). $v \in \mathcal{V}$.

$SP_{k,n}^{v,m,t}$ Energy sales price (EUR/kWh, stochastic, [enerSellPrice:decimal]). $k \in \mathcal{K}$, $n \in \mathcal{N}_S^k$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. For the types of energy that can be sold, there is a price for each operational period.

SU_i^v Subsidies for a technology (EUR/kW, stochastic, [techSubsid:decimal]). $i \in \mathcal{I}$, $v \in \mathcal{V}$. Policy makers can subsidize the investment of some efficient technologies. Usually an amount per kW is paid.

XZ_i^a Existing devices (Devices/kW/kWh, deterministic, [techEdDevices:decimal/integer]). $i \in \mathcal{I}$, $a \in \mathcal{A}$. Number of existing devices of a given age of each technology at the start of the optimization horizon of.

2.7 Decision Variables

$e^{v,m,t}$ Primary energy consumed per operational period (kWh, Operational, [enerConsumed:decimal]). $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. This is a computed variable for the energy consumption of the building during each short-term period.

$h_{k,n}^v$ Tariff choice (unitless, Strategic, [marketChoice:binary]). $k \in \mathcal{K}$, $n \in \mathcal{N}_{Tr}^k$, $v \in \mathcal{V}$. This is the decision for selecting among different tariffs. The choice is done for the subsequent period.

$r_{i,k}^{v,m,t}$ Energy stored (kWh, Operational, [stoEnerStored:decimal]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. This is an inventory of the amount of energy that is stored in the energy-storage technologies during each short-term period. It is calculated using the operational decisions and the technology parameters.

$ri_{i,k}^{v,m,t}$ Energy input to storage (kWh, Operational, [stoInput:decimal]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. Addition to energy storage for each type of energy during each short-term period.

$ro_{i,k}^{v,m,t}$ Energy output from storage (kWh, Operational, [stoOutput:decimal]). $i \in \mathcal{I}_{Sto}$, $k \in \mathcal{K}_{Po}^i$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. Release from each energy-storage technology of each type of energy during each operational period.

$u_{k,n}^{v,m,t}$ Energy to purchase under a given tariff (kWh, Operational, [marketPurchEner:decimal]). $k \in \mathcal{K}$, $n \in \mathcal{N}_{Pur}^k$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. Energy purchased in the market for each type of energy, to be delivered during each operational period.

$w_{k,n}^{v,m,t}$ Energy to sell under a given tariff (kWh, Operational, [marketSellEner:decimal]). $k \in \mathcal{K}$, $n \in \mathcal{N}_S^k$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. Energy of each type of energy to be sold in the market during each operational period.

$x_i^{v,a}$ Installed units of a given age for each technology and node (Devices/kW/kWh, Strategic, [techInstalled:integer/decimal]). $i \in \mathcal{I}$, $v \in \mathcal{V}$, $a \in \mathcal{A}$. This is a computed variable.

xc_i^v Available capacity of a technology at each node (kW or kWh (storage), Strategic, [techAvailCap:decimal]). $i \in \mathcal{I}$, $v \in \mathcal{V}$. This capacity is computed through the decisions and the parameters.

$xd_i^{v,a}$ Number of units of a technology to be decommissioned (Devices or kW, Strategic, `[techDecom:integer/decimal]`). integer for $i \in \mathcal{I}_{Ds}$, $v \in \mathcal{V}$, $a \in \mathcal{A}_{Old}$; continuous for $i \in \mathcal{I}_{Cn}$, $v \in \mathcal{V}$, $a \in \mathcal{A}_{Old}$. For continuously-sized technologies, this is the total capacity to be decommissioned. For discretely-sized technologies, it denotes number of devices to decommission.

xi_i^v Number of units of a technology to be installed (Devices or kW, Strategic, `[techInst:integer/decimal]`). integer for $i \in \mathcal{I}_{Ds}$, $v \in \mathcal{V}$; continuous for $i \in \mathcal{I}_{Cn}$, $v \in \mathcal{V}$. For discretely-sized technologies, this is an integer variable, whilst for continuously-sized technologies, it is a continuous one.

$y_{i,k}^{v,m,t}$ Energy generator input (kWh, Operational, `[genInputEner:decimal]`). $i \in \mathcal{I}_{Gen}$, $k \in \mathcal{K}_{In}^i$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. Amount of energy used as input to an energy-creating technology, for each type of energy, operational profile, and period.

$z_{i,k}^{v,m,t}$ Energy generator output (kWh, Operational, `[genOutputEner:decimal]`). $i \in \mathcal{I}_{Gen}$, $k \in \mathcal{K}_{Out}^i$, $v \in \mathcal{V}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_{Tm}^m$. Amount of energy as output from an energy-creating technology, for each type of energy, operational profile, and period.

2.8 Equations

(1) Strategic

Available new technologies (devices) at each node

The available new devices of a technology (age zero) are equal to the ones installed at each node.

$$x_i^{v,0} = xi_i^v \quad \forall\, i \in \mathcal{I},\ v \in \mathcal{V} \tag{1}$$

Available old technologies (devices) at future nodes

The number of available devices whose age is not zero is equal to the number available at the previous node minus the number of decommissioned ones.

$$x_i^{v,a} = x_i^{Pa(v),a-1} - xd_i^{v,a} \quad \forall\, i \in \mathcal{I},\ v \in \mathcal{V}_{Fut},\ a \in \mathcal{A}_{Ages}^{i,v} \cap \mathcal{A}_{Old} \tag{2}$$

Available old technologies (devices) at root node

For technologies existing before the start of the optimization horizon, the number of devices available at the root note is equal to the number of existing devices minus those decommissioned at the beginning of the first period.

$$x_i^{1,a} = XZ_i^{a-1} - xd_i^{1,a} \quad \forall\, i \in \mathcal{I},\ a \in \mathcal{A}_{Ages}^{i,v} \cap \mathcal{A}_{Old} \tag{3}$$

Technology capacity calculation

The total capacity of a technology is the sum of the capacities of the installed devices at any age, corrected by the aging factors and nominal capacity.

$$xc_i^v = G_i \cdot \sum_{a \in \mathcal{A}_{Ages}^{i,v}} AG_i^a \cdot x_i^{v,a} \quad \forall\, i \in \mathcal{I},\ v \in \mathcal{V} \tag{4}$$

Investment limit

An upper limit may be imposed on the total installation, decommissioning, and maintenance cost.

$$\sum_{i \in \mathcal{I}} \left((CI_i^v - SU_i^v) \cdot G_i \cdot xi_i^v + \sum_{a \in \mathcal{A}_{Ages}^{i,v}} CD_i^{v,a} \cdot G_i \cdot xd_i^{v,a} + \sum_{a \in \mathcal{A}_{Ages}^{i,v}} CM_i^{v,a} \cdot G_i \cdot x_i^{v,a} \right) \le IL^v \quad \forall\, v \in \mathcal{V} \tag{5}$$

Purchase tariff choice

Only one purchase tariff is allowed.

$$\sum_{n \in \mathcal{N}^k_{Pur}} h^v_{k,n} = 1 \quad \forall\, v \in \mathcal{V},\;\; k \in \mathcal{K}_{EP} \tag{6}$$

Sales tariff choice

Only one sales tariff is allowed.

$$\sum_{n \in \mathcal{N}^k_{S}} h^v_{k,n} = 1 \quad \forall\, v \in \mathcal{V},\;\; k \in \mathcal{K}_{ES} \tag{7}$$

Physical limit

There is a limit for installing technologies, usually established by the space available in the site. Note that within the optimizer, it can be implemented as a variable upper limit rather than a constraint.

$$\sum_{a \in \mathcal{A}^{i,v}_{Ages}} x^{v,a}_i \leq LP^v_i \quad \forall\, i \in \mathcal{I},\; v \in \mathcal{V} \tag{8}$$

Required efficiency

The amount of energy consumed and sold must be larger than the amount of primary energy consumed corrected by the efficiency parameter.

$$\sum_{m \in \mathcal{M}} DM^m \cdot \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}^m_{Tm}} \left(D^{v,m,t}_k + \sum_{n \in \mathcal{N}^k_S} w^{v,m,t}_{k,n} \right) \geq EF^v \cdot \sum_{m \in \mathcal{M}} DM^m \cdot \sum_{t \in \mathcal{T}^m_{Tm}} e^{v,m,t} \quad \forall\, v \in \mathcal{V} \tag{9}$$

Emissions limit

The total emissions of each considered pollutant cannot exceed a specified limit.

$$\sum_{m \in \mathcal{M}} DM^m \cdot \sum_{t \in \mathcal{T}^m_{Tm}} \left(\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}^i_{In}} LH^v_{i,k,l} \cdot y^{v,m,t}_{i,k} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}^k_{Pur}} LC^v_{k,l,n} \cdot u^{v,m,t}_{k,n} \right) \leq PL^v_l \quad \forall\, l \in \mathcal{L},\; v \in \mathcal{V} \tag{10}$$

(2) Operational

Output energy calculation

The amount of output energy is calculated from the input energy and the conversion factor.

$$z^{v,m,t}_{i,k'} = \sum_{k \in \mathcal{K}^i_{In}} EC^v_{i,k,k'} \cdot y^{v,m,t}_{i,k} \quad \forall\, v \in \mathcal{V},\; m \in \mathcal{M},\;\; i \in \mathcal{I}_{Gen},\; k' \in \mathcal{K}^i_{Out}, t \in \mathcal{T}^m_{Tm} \tag{11}$$

Technology output limit

The energy that can be supplied by a technology is constrained by the availability of the technology and its capacity.

$$z^{v,m,t}_{i,k} \leq DT^m \cdot AF^{v,m,t}_i \cdot xc^v_i \quad \forall\, v \in \mathcal{V},\; m \in \mathcal{M},\;\; i \in \mathcal{I}_{Gen},\; t \in \mathcal{T}^m_{Tm}, k \in \mathcal{K}^i_{Po} \tag{12}$$

Storage available

The energy stored in each period is the energy stored in the previous period, plus the energy sent to storage, minus the energy released from storage. All flows are corrected by their respective loss ratio parameter.

$$r^{v,m,t}_{i,k} = OS_{i,k} \cdot r^{v,m,t-1}_{i,k} + OI^v_{i,k} \cdot ri^{v,m,t-1}_{i,k} - OO^v_{i,k} \cdot ro^{v,m,t-1}_{i,k} \quad \forall\, v \in \mathcal{V},\; m \in \mathcal{M},\;\; i \in \mathcal{I}_{Sto},\; t \in \mathcal{T}^m_{Tm}, k \in \mathcal{K}^i_{Po} \tag{13}$$

Storage release limit

The amount of energy that can be discharged from any energy-storage technology is limited by the installed capacity and the maximum discharge rate.

$$ro_{i,k}^{v,m,t} \leq OX_{i,k}^{v} \cdot DT^{m} \cdot xc_{i}^{v} \quad \forall\, v \in \mathcal{V},\ m \in \mathcal{M},\ \ i \in \mathcal{I}_{Sto},\ t \in \mathcal{T}_{Tm}^{m}, k \in \mathcal{K}_{Po}^{i} \tag{14}$$

Storage charge limit

The amount of energy that can be charged to any energy-storage technology is limited by the installed capacity and the maximum charge rate.

$$ri_{i,k}^{v,m,t} \leq OY_{i,k}^{v} \cdot DT^{m} \cdot xc_{i}^{v} \quad \forall\, v \in \mathcal{V},\ m \in \mathcal{M},\ \ i \in \mathcal{I}_{Sto},\ t \in \mathcal{T}_{Tm}^{m}, k \in \mathcal{K}_{Po}^{i} \tag{15}$$

Storage level between periods

The storage level at the first short-term period must be equal to the level at the final period (in the same strategic period).

$$r_{i,k}^{v,m,1} = r_{i,k}^{v,m,Tmax} \quad \forall\, v \in \mathcal{V},\ m \in \mathcal{M},\ \ i \in \mathcal{I}_{Sto},\ k \in \mathcal{K}_{Po}^{i} \tag{16}$$

Lower storage limit

The amount of energy that can be stored in any energy-storage technology must be greater than the capacity installed, corrected by the minimum charge required.

$$r_{i,k}^{v,m,t} \geq OA_{i,k}^{v} \cdot xc_{i}^{v} \quad \forall\, v \in \mathcal{V},\ m \in \mathcal{M},\ \ i \in \mathcal{I}_{Sto},\ t \in \mathcal{T}_{Tm}^{m}, k \in \mathcal{K}_{Po}^{i} \tag{17}$$

Upper storage limit

The amount of energy that can be stored in any energy-storage technology must be lower than the capacity installed, corrected by the maximum charge allowed.

$$r_{i,k}^{v,m,t} \leq OB_{i,k}^{v} \cdot xc_{i}^{v} \quad \forall\, v \in \mathcal{V},\ m \in \mathcal{M},\ \ i \in \mathcal{I}_{Sto},\ t \in \mathcal{T}_{Tm}^{m}, k \in \mathcal{K}_{Po}^{i} \tag{18}$$

Energy balance

The energy supplied must meet the energy demand minus the energy saved due to passive technologies. It is composed of the energy produced with energy-creating technologies plus the energy purchased in the market minus the energy sold, energy for storage and energy for production. On the demand side, the energy saved with passive technologies diminish the total demand.

$$\begin{aligned}
&\sum_{i \in \mathcal{I}_{Gen}} z_{i,k}^{v,m,t} - \sum_{i \in \mathcal{I}_{Gen}} y_{i,k}^{v,m,t} + \sum_{n \in \mathcal{N}_{Pur}^{k}} u_{k,n}^{v,m,t} - \sum_{n \in \mathcal{N}_{S}^{k}} w_{k,n}^{v,m,t} \\
&+ \sum_{i \in \mathcal{I}_{Sto}} \left(ro_{i,k}^{v,m,t} - ri_{i,k}^{v,m,t} \right) = D_{k}^{v,m,t} \cdot \left(1 - \sum_{i \in \mathcal{I}_{PU}} OD_{i,k}^{v} \cdot xc_{i}^{v} \right) \\
&\qquad \forall\, k \in \mathcal{K},\ v \in \mathcal{V},\ m \in \mathcal{M},\ t \in \mathcal{T}_{Tm}^{m}
\end{aligned} \tag{19}$$

Purchasing limit by contract

The amount of energy that can be purchased at a given node must not exceed the amount stipulated in the previously signed contract.

$$u_{k,n}^{v,m,t} \leq h_{k,n}^{v} \cdot ME_{k,n} \cdot DT^{m} \quad \forall\, k \in \mathcal{K},\ m \in \mathcal{M},\ n \in \mathcal{N}_{Pur}^{k}, t \in \mathcal{T}_{Tm}^{m} \tag{20}$$

Sales limit by contract

The amount of energy that can be purchased at a given node must not exceed the amount stipulated in the previously signed contract.

$$w_{k,n}^{v,m,t} \leq h_{k,n}^{v} \cdot ME_{k,n} \cdot DT^{m} \quad \forall\, k \in \mathcal{K},\ m \in \mathcal{M},\ n \in \mathcal{N}_{S}^{k}, t \in \mathcal{T}_{Tm}^{m} \tag{21}$$

Primary energy calculation

The primary energy (not from a fictitious market) consumed is the sum of the processed energy of each type and the energy used as an input fuel.

$$e^{v,m,t} = \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}^k_{Pur}} B_{k,n} \cdot u^{v,m,t}_{k,n} \quad \forall\, v \in \mathcal{V},\ m \in \mathcal{M},\ t \in \mathcal{T}^m_{Tm} \tag{22}$$

(3) Objective

Total discounted expected cost

The total cost consists of the installation cost (the term within the summation over i in (23a)), the decommissioning cost (first term in (23b)), the maintenance cost (second term in (23b)), the energy purchasing cost (first term in (23c)), and the operation cost (23d). Incomes from energy sales are subtracted in (23c) and subsidies directly diminish the technology installation costs in (23a).

$$\text{minimize} \sum_{v\in\mathcal{V}} (1+DR)^{-PT^v} \cdot PR^v \cdot \left(\sum_{i\in\mathcal{I}} \left((CI^v_i - SU^v_i) \cdot G_i \cdot xi^v_i \right.\right. \tag{23a}$$

$$\left. + \sum_{a\in\mathcal{A}^{i,v}_{Ages}} CD^{v,a}_i \cdot G_i \cdot xd^{v,a}_i + \sum_{a\in\mathcal{A}^{i,v}_{Ages}} CM^{v,a}_i \cdot G_i \cdot x^{v,a}_i \right) \tag{23b}$$

$$+ \sum_{m\in\mathcal{M}} DM^m \cdot \sum_{t\in\mathcal{T}^{m,t}_{Tm}} \left(\sum_{k\in\mathcal{K}_{EP}} \sum_{n\in\mathcal{N}^{k,n}_{Pur}} PP^{v,m,t}_{k,n} \cdot u^{v,m,t}_{k,n} - \sum_{k\in\mathcal{K}_{\mathcal{ES}}} \sum_{n\in\mathcal{N}^{k,n}_S} SP^{v,m,t}_{k,n} \cdot w^{v,m,t}_{k,n} \right. \tag{23c}$$

$$\left.\left. + \sum_{i\in\mathcal{I}_{Gen}, k\in\mathcal{K}^{i,k}_{Out}} CO^v_{i,k} \cdot z^{v,m,t}_{i,k} + \sum_{i\in\mathcal{I}_{Sto}, k\in\mathcal{K}^{i,k}_{Po}} CO^v_{i,k} \cdot r^{v,m,t}_{i,k} \right)\right) \tag{23d}$$